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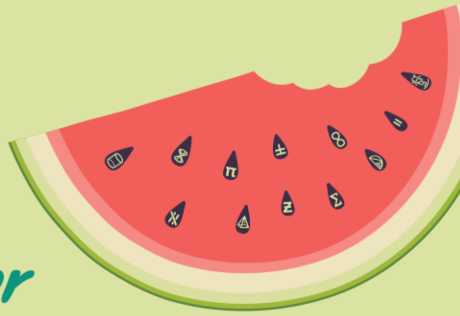

Tempo-Spatial Analysis of Australian Fuel Price Dynamics

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Contents

Abstract

Fuel prices in Australian capital cities exhibit cyclical patterns, where short-term pricing strategies often deviate from broader global trends. Although macroeconomic variables such as crude oil prices and exchange rates strongly influence overall price levels, the short-term fluctuations recorded in retail fuel markets are largely shaped by competitive interactions between local retailers. This research applies a combination of residual analyses and spatial econometric approaches to examine how pricing decisions ripple across neighbouring markets. The findings indicate that the pricing behaviour of fuel retailers aligns with the Maskin-Tirole dynamic oligopoly model, characterised by Edgeworth cycles of incremental undercutting and sudden resets. Furthermore, spatial analysis reveals meaningful price interdependence among nearby stations. By distinguishing localised competitive dynamics from overarching economic forces, this study illuminates the tempo-spatial nature of fuel price setting and discusses its implications for market behaviour.

1 Introduction

Fuel prices in Australian capital cities follow a cyclical pattern, featuring regular price fluctuations that often occur independently of long-term macroeconomic trends [Com25]. While broader trends in fuel prices are generally influenced by global crude oil prices, exchange rates, and inflation, the short-term oscillations observed in retail pricing are frequently driven by strategic interactions among competing retailers. Previous research indicates that these price cycles exhibit asymmetry, with prices rising sharply but declining more gradually, a feature consistent with the Edgeworth cycles postulated in the Maskin-Tirole dynamic oligopoly model [MT88].

Statement of Authorship

All code development, data analysis, and model implementations presented in this report were conducted by the author. Guidance regarding the research trajectory, as well as the interpretation of results, was provided by my supervisor, Dr. Dietmar Oelz. Moreover, while most algorithms were written from scratch, I used the `find_peaks_cwt` function from the SciPy library [Sci] to perform the continuous wavelet transform (CWT) peak detection, rather than implementing a custom version of the CWT algorithm.

1.1 Motivation

Understanding these cyclical pricing behaviours is critical for both economic theory and regulatory oversight. Some studies [Val10; Wan08] suggest that asymmetric price adjustments may point to market inefficiencies or tacit coordination among fuel retailers. Additionally, spatial factors play a major role in determining fuel prices, given that no service station operates in isolation; rather, its pricing decisions are influenced by nearby competitors. Research by [BCI20] finds considerable spatial interdependence in fuel prices, reinforcing the importance of local competition.

The aim of this study is to devise a modelling framework that explicitly incorporates **spatial dependencies** in retail fuel pricing. Specifically, the objectives are to:

- Investigate the **tempo-spatial** characteristics of fuel price cycles.
- Test for **spatial dependence** using Moran's I statistic [Mor50].
- Evaluate how pricing behaviour aligns with the **Maskin-Tirole dynamic oligopoly model**.

Through rigorous empirical models, this research provides insights into retailer pricing strategies and assists policymakers in identifying potential collusive practices within fuel markets.

1.2 Previous Works

Research on fuel pricing dynamics has historically centred on two main dimensions: (1) the **temporal** element, examining how prices evolve over time and whether cycles follow discernible patterns, and (2) the **spatial** element, exploring how one location's prices influence prices at neighbouring sites. The following subsections review representative studies in these areas.[1]

1.2.1 Temporal Pricing Dynamics and Asymmetry

One of the earliest investigations of Australian retail fuel price cycles was carried out by Valadkhani [Val10], who analysed both long- and short-term determinants in capital cities. Valadkhani showed that macroeconomic factors such as crude oil prices and exchange rates predominantly drive long-run price trajectories. However, he also found evidence of **asymmetric price adjustments**, whereby prices rise quickly but take longer to fall, suggesting the possibility of non-competitive behaviour.

Building on this, Wang [Wan08] examined whether these cycles could be explained by collusive communication among Australian fuel retailers. Using data from an Australian Competition and Consumer Commission (ACCC) investigation into price fixing in Ballarat, Wang demonstrated that the retail price cycles observed are broadly consistent with the **Maskin-Tirole (MT) model** of dynamic oligopoly. This finding supports the idea that price cycles are driven, at least in part, by coordinated behaviour among retailers.

1.2.2 Spatial Dependence in Fuel Pricing

While much of the early work emphasised temporal aspects of pricing cycles, attention has increasingly turned to the **spatial correlation** among retailers. In one key study, Bergantino, Capozza, and Intini [BCI20] used spatial econometric techniques to explore how geographic proximity influences retail fuel prices in Rome, Italy, finding clear evidence that service stations closely monitor and react to each other's prices.

Focusing on South East Queensland, Hogg et al. [Hog+12] investigated the impact of **vertical restraints and branding** on retail fuel prices. Their results showed that **branded fuel stations** typically charge higher

[1] A detailed summary of past works, including key findings in tabular form, is provided in **Appendix 6.3**.

prices, while the presence of independent retailers in a locality exerts downward pressure on prices, thereby enhancing competition. These outcomes highlight the role of both **market structure** and the local competitive environment.

2 Maskin-Tirole (MT) Model

Originally proposed by Maskin and Tirole [MT88], the Maskin-Tirole (MT) model offers a theoretical framework for *dynamic* price competition in oligopolistic markets (often duopolies). At its core, the model assumes that firms use *Markov Perfect Equilibria*, implying that each firm's current price choice depends only on the "state" of the system—commonly captured by the most recent prices set by each firm. By modelling this state-dependent decision process, the MT model demonstrates how firms can engage in cyclical price-setting behaviour, known as *Edgeworth cycles*.

2.1 Key Features of the MT Model

- **High-price (reset) phase:** Firms temporarily coordinate on higher prices to earn larger margins.
- **Undercutting phase:** A firm lowers its price marginally below its rival's to capture additional market share.
- **Price-war phase:** Continuing undercuts drive prices down, eroding profit margins for all.
- **Return to high-price phase:** Once prices become unprofitable, a *reset* event occurs, with at least one firm attempting to jump back to a higher price.

2.2 Two-Player Game

In a two-firm scenario, the MT model typically exhibits clearer cycles, as depicted in Figure 1. The strategic logic is relatively straightforward:

1. **High-price phase:** Both firms stick to a higher price to enjoy larger margins.
2. **Undercutting phase:** One firm drops its price slightly to gain a larger share of sales.
3. **Price war:** The other firm responds with a further reduction, triggering a downward price race.
4. **Reset:** Eventually, one firm finds it profitable to spike its price back to a higher level, hoping the other firm will follow.

Because only two firms are involved, each firm's best response is relatively direct—match or undercut the rival's price. This dynamic often leads to the repeating "peak-to-trough" cycles illustrated in Figure 1. **Wang2009** further shows that observed fuel pricing cycles, even under government restrictions in Perth, align well with these theoretical Edgeworth cycles.

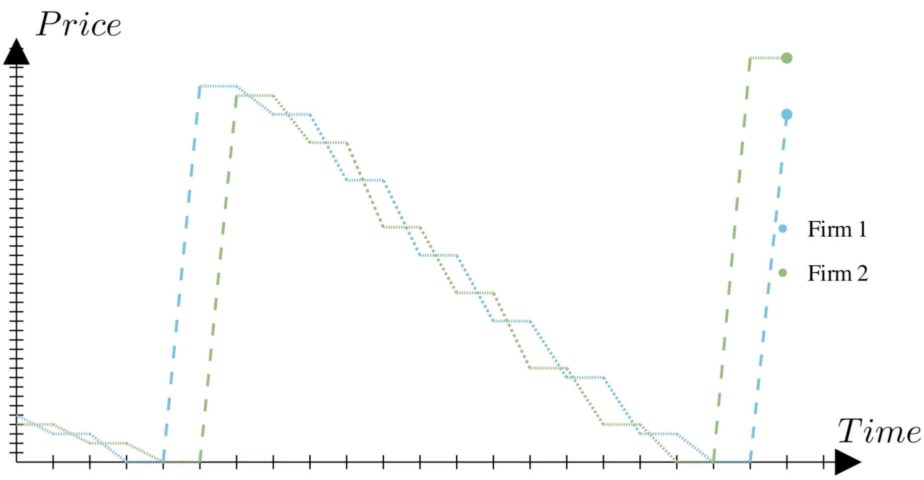

Figure 1: Edgeworth cycle in a two-player MT model.

2.3 Three-Player Game

When extended to three or more firms, the MT model retains the same general structure but becomes more challenging to sustain due to coordination issues. Figure 2 shows a typical three-firm cycle attempt. The main complications are:

- **Spike Attempt:** A single firm may jump to a higher price, but the others have an immediate incentive to remain low and capture market share.
- **Coordination Problem:** Without explicit or tacit agreement, there is no guarantee that all firms will "spike" at the same time.
- **Outcome:** Many attempts to reset prices fail, resulting in a continuation of the price war unless firms can reliably coordinate the high-price phase.

Overall, the MT model predicts that cyclical pricing can persist in an oligopolistic market if firms repeatedly reset prices and then undercut each other's high prices. While such cycles emerge more consistently with two firms, they can also occur with three or more firms provided that some mechanism—explicit or otherwise—helps resolve the coordination problem of resetting to higher prices. This insight has proved particularly relevant in the study of retail fuel markets, where evidence of Edgeworth cycles abounds.

3 Dataset

This section outlines the sources of our fuel price data, provides an overview of its structure and magnitude, and then discusses how we remove global price trends to isolate cyclical pricing (residual) components. We also present the continuous wavelet transform (CWT) methodology used to detect local minima and derive a baseline.

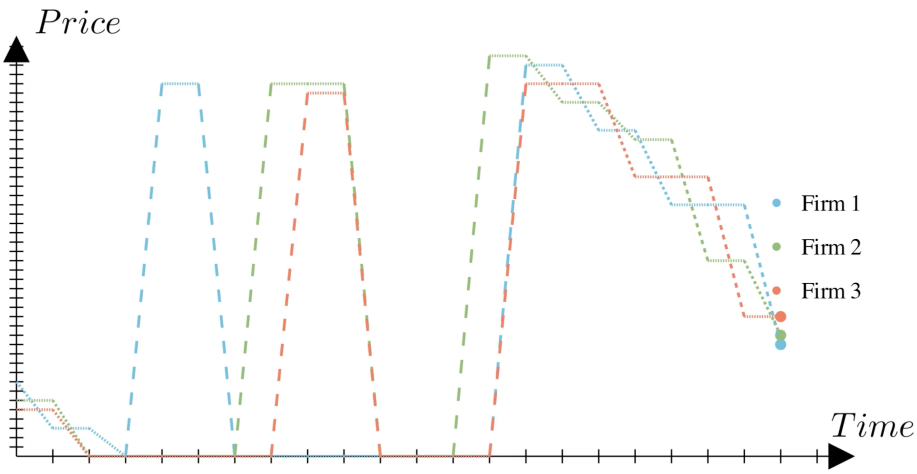

Figure 2: Attempted Edgeworth cycle in a three-player MT model.

3.1 Sources of the Data

Retail fuel prices were collected from three Australian state-level reporting schemes:

- **FuelWatch (WA)** [Gov25], which mandates daily price notification;
- **FuelCheck (NSW)** [NSW25], requiring real-time reporting of price changes;
- **Fuel Price Reporting Scheme (QLD)** [Tre], obliging outlets to lodge new prices whenever they adjust them.

These legislative initiatives arose to boost consumer transparency and discourage collusive behaviour. While South Australia also enforces mandatory reporting, historical data were not freely accessible in a convenient form.

3.2 Data Description

Our combined dataset spans **7,859 unique service stations** across Queensland, New South Wales, and Western Australia, with **3,971,954 individual records of price changes**. Figure 3 shows an example of the *raw, unprocessed* price series for a single site (Site 10103).

Since this is an *event-driven* dataset (a new entry is recorded only when a price changes), timestamps for one site do not necessarily align with others. To simplify indexing and modelling, we create a uniform minute-level grid:

$$t = 0 \quad \longleftrightarrow \quad \text{2018-01-12 00:00},$$

incrementing t by 1 for each minute up to 2024-12-28 12:00. If a site does not change its price in a given minute, we either carry its previous value forward or omit that minute if focusing on change events alone.

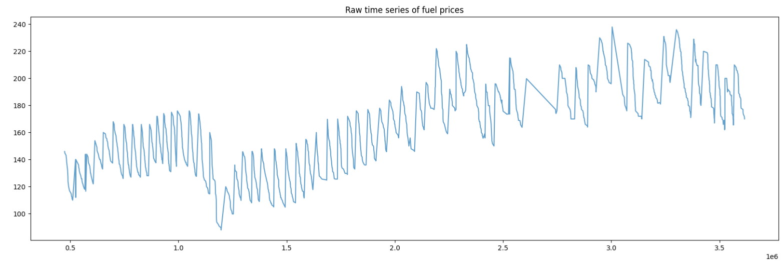

Figure 3: Raw unprocessed price data for a sample site (Site 10103). Multiple changes may occur in one day or none at all.

3.3 Global Trends and Motivation for Detrending

Fuel prices reflect both *macro* (global) and *micro* (local cyclical) components. Large-scale changes—attributed to inflation, exchange-rate shifts, or global supply factors—can obscure the local "price war" dynamics we seek to model [**kpodar2016**; Com25]. Figure 4 shows an example of raw data overlaid with a polynomial spline baseline. The cyclical "peaks and troughs" persist despite global price-level fluctuations.

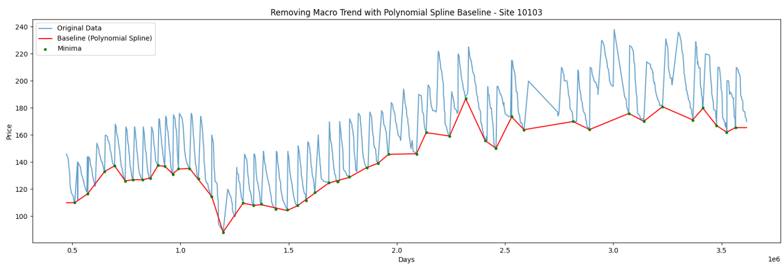

Figure 4: Raw data (blue) for a sample site, with a polynomial spline baseline (red). Extracting the local (cyclical) component requires detrending the series.

Distinguishing these cyclical price changes from broader trends is crucial for analysing whether the short-run dynamics align with Maskin-Tirole Edgeworth cycles. In practice, this means defining a *residual*:

$$r(t) \;=\; p(t) - b(t),$$

where $p(t)$ is the observed retail price and $b(t)$ is a smoothly varying baseline function capturing macro-level effects.

3.4 Residual Definition and Baseline Approaches

We explored multiple methods to construct the baseline $b(t)$. Figure 5 shows an example of the residual series $r(t)$ when using a polynomial spline. The baseline ensures that long-run price swings are separated out, leaving short-term fluctuations in the residuals.

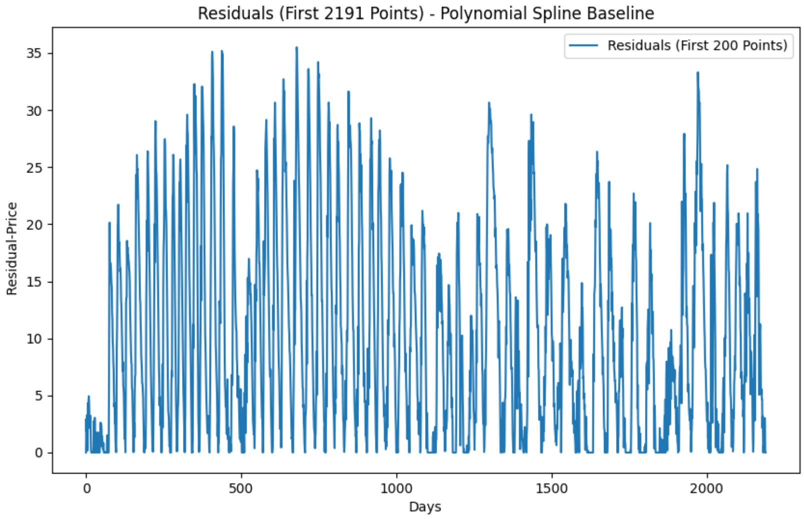

Figure 5: Residual series (blue) after subtracting a baseline fit (red) from raw data.

Savitzky-Golay Filter. A Savitzky-Golay filter [**savitzky1964smoothing**] uses locally fitted polynomials to smooth the series. It is simple to tune with window size and polynomial order, but it can *overfit* if the window is too small or the polynomial order is too high, inadvertently capturing the very price cycles we want in the residual. Hence, it can flatten out true cyclical activity.

Wavelet-Based Baseline. As a more robust alternative, we adopt a *continuous wavelet transform* approach that identifies local minima in the negative of the price series. This yields a piecewise baseline that remains close to cyclical "valleys" while ignoring small short-lived deviations, thus avoiding overfitting. The resulting residual has an interpretable lower bound of 0, meaning $r(t) \geq 0$ indicates how far the price sits above the local trough.

3.5 Wavelet-Based Peak-Finding Method

Motivation. The continuous wavelet transform (CWT) approach is well suited to time series that may be *heteroskedastic*, *non-stationary*, or irregularly sampled—properties that characterise our event-driven data [DKL06; Bat93; Sci]. By examining how the wavelet coefficients vary across scales, we can robustly identify local maxima or minima even when the amplitude and spacing of price cycles change over time.

3.5.1 Continuous Wavelet Transform (CWT) Overview

For a function $f(t)$ and chosen wavelet ψ, the CWT is defined by:

$$W(a,b) \;=\; \int_{-\infty}^{\infty} f(t)\,\psi\Big(\frac{t-b}{a}\Big)\,dt,$$

where $a > 0$ is the scale parameter and b is the translation parameter [Bat93]. Large wavelet coefficients at certain (a, b) indicate salient features (peaks/troughs) in $f(t)$.

3.5.2 Algorithm for Identifying Ridges

While we discuss the general idea of *ridge-finding* here, the detailed pseudocode is given in **Appendix 6.3**. Briefly, local maxima are identified at the coarsest wavelet scale and then tracked across decreasing scales, forming continuous ridge lines. By applying this to the $-p(t)$ series, we find local minima in the actual price data.

3.6 Comparison and Conclusion on Baseline Approaches

Figure 6 shows a polynomial spline baseline (red) along with the original raw data (blue). Overfitting can cause the spline to "wiggle" too closely to short-run price dips, inadvertently cancelling out legitimate cycle depths in the residual. In contrast, wavelet-based detection (Figure 7) maintains a smoother baseline, preserving the amplitude of cyclical price changes in the residual series.

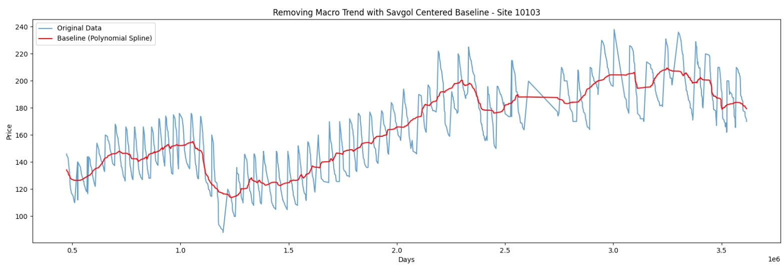

Figure 6: Raw data (blue) with a polynomial spline baseline (red). Overly flexible baselines can remove true cyclical patterns.

Essentially, while the Savitzky-Golay filter is a well-known smoothing technique, its susceptibility to overfitting makes it less suitable for consistently isolating local fuel price cycles. The wavelet-based approach offers clearer interpretability (with $r(t) \geq 0$ marking how far the price sits above the local trough) and better consistency across thousands of stations. Consequently, we adopt the wavelet (peak-finding) algorithm to compute baselines and residuals, which then form the foundation of our subsequent modelling.

4 Statistical Tests

4.1 Testing for Spatial Dependence (Moran's I)

In line with Bergantino, Capozza, and Intini [BCI20], we investigate whether the residual pricing series across different service stations displays *spatial dependence*. Specifically, we use **Moran's I statistic** [Mor50], a widely employed measure in spatial econometrics that quantifies the degree of similarity among nearby observations.

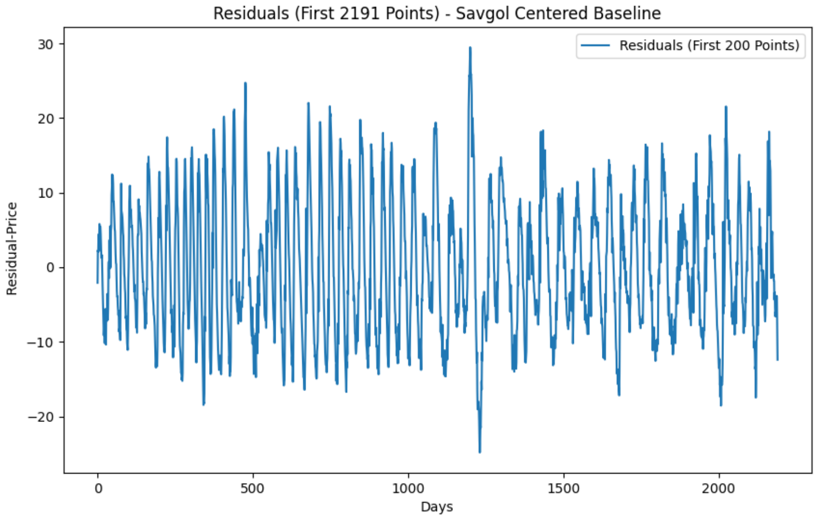

Figure 7: Residuals after subtracting the spline baseline from raw data. Overfitting risks flattening the short-run cycles we aim to capture.

Definition and Interpretation. Let y_i denote the residual price for station i, and let $\bar{y}$ be the mean residual across all N stations. Suppose w_{ij} represents a spatial weight capturing the proximity of station j to station i. Moran's I is given by

$$I = \frac{N \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} \, (y_i - \bar{y})(y_j - \bar{y})}{\left(\sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} \right) \sum_{i=1}^{N} (y_i - \bar{y})^2}.$$

Its theoretical range is $[-1, 1]$, with:

- $I = -1$ indicating **perfect negative** spatial correlation,
- $I = 0$ indicating **no** spatial correlation,
- $I = +1$ indicating **perfect positive** spatial correlation.

A discussion of the specific Haversine distance formula used to define w_{ij} can be found in **Appendix 6.3**.

Result and Discussion. With our dataset of N stations, we obtain a Moran's I of **0.889**, implying strong positive spatial correlation in fuel price residuals. In practical terms, this suggests that *nearby stations* tend to exhibit highly similar short-run price deviations from their baselines. Such a result is unsurprising, given the known clustering of competitor prices in local areas. Figure 8 provides a qualitative illustration for five closely located sites over a five-month period. Notice how their prices move in near lockstep, forming a repeated "peak and trough" structure reminiscent of the Edgeworth cycles described in §2.

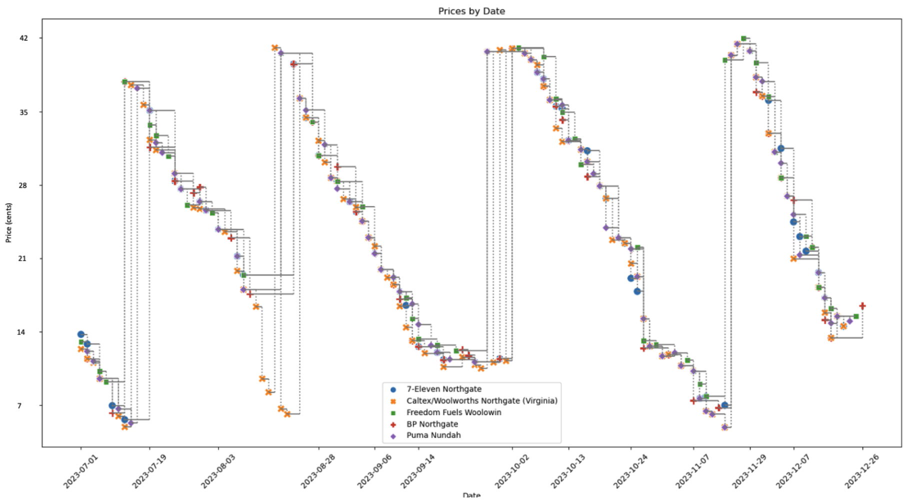

Figure 8: Prices at five neighbouring sites across a five-month period. The high degree of spatial correlation is evident.

4.2 Brief Note on Stationarity

Prior literature strongly suggests that fuel price cycles are *stationary* in their residual form [DF79]. Our own checks using the Augmented Dickey-Fuller test (full details in **Appendix 6.3**) confirm that for all sites, the p-values are sufficiently small to reject the null of a unit root. This indicates that the cyclical series is stationary, reinforcing the reliability of subsequent analyses.

5 Approximate Modelling Methods

In order to capture the short-run dynamics of local price competition, we tested two approximate models on the residual series: a purely **spatial** model and a **game-based** model loosely inspired by the MT framework. Both rely on the strong spatial dependence identified in §**??**, yet they differ in how they handle control or "reset" behaviour.

5.1 Spatial Model

Our *spatial model* uses a simple nearest-neighbour averaging rule. Specifically, let $p_i(t)$ be the residual price at site i and discrete time t. Then,

$$p_i(t+1) \;=\; \frac{1}{n} \sum_{j \in \mathcal{N}_i} p_j(t),$$

where $\mathcal{N}_i$ is the set of the n nearest sites to station i (excluding i itself), and $n = 10$. Since Moran's I indicates strong spatial correlation among neighbouring stations, we expect this model to be a reasonable first approximation. However, it lacks any mechanism for "resetting" or "undercutting," so it might fail to capture the cyclical extremes characteristic of Edgeworth cycles.

5.2 Game-Based Model

We also developed a *game-based model* more closely aligned with the Maskin-Tirole perspective. Let $x = p_i(t)$ be station i's residual price at time t. We look at the 10 closest stations $\mathbf{locals} = \{p_j(t)\}$ and define:

$$f(x) = \begin{cases} \mathbb{I}\big(\max(\mathbf{locals}) - x > 30\big)\,\big[\max(\mathbf{locals}) - \varepsilon\big] \\ \quad + \mathbb{I}\big(\max(\mathbf{locals}) - x < 30\big)\,\min\big(x,\ \overline{\mathbf{locals}} - \varepsilon\big), & \text{if } 0 < x < 2, \\ \min\big(x,\ \overline{\mathbf{locals}} - \varepsilon\big), & \text{if } x \geq 2, \\ 0, & \text{if } x < 0, \end{cases}$$

where

$$\varepsilon = |\xi|, \quad \xi \sim \mathcal{N}(0,1),$$

and $\overline{\mathbf{locals}}$ is the (local) average of neighbouring stations' prices. In essence, this function attempts large upward "jumps" if rivals' maxima greatly exceed x, but drives x down to the local average (minus a small random offset) if prices are already near or above the top of local competition. Unlike the purely spatial model, this scheme includes an explicit notion of "reset" and "undercutting," reflecting the cyclical dynamics of Edgeworth competition.

5.3 Forward Simulation and Results

Figure 9 shows a short sample (the first 30% of the dataset) where both models are forward-simulated against the true residual series (blue) and an unseen portion of data used for testing (grey). Although the entire timeline is more comprehensive, the full plot can be hard to interpret visually, so we provide the complete simulation in **Appendix 6.3** (Figure 10) for reference.

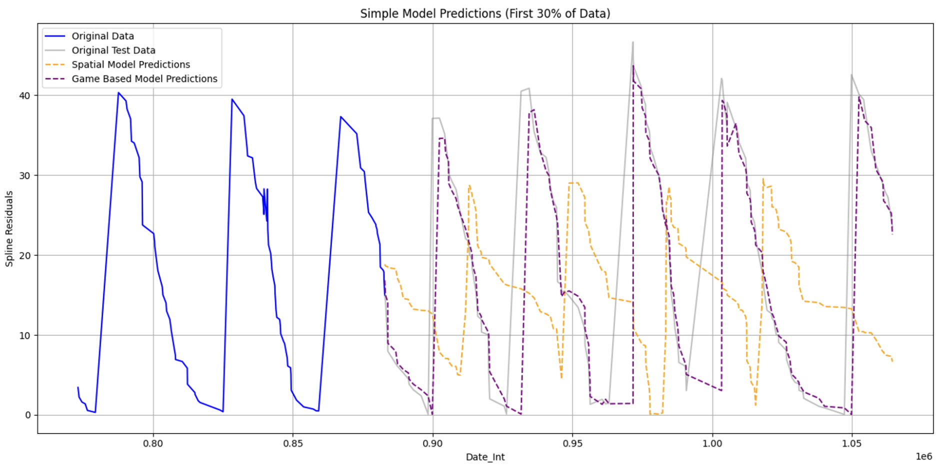

Figure 9: Sample of forward simulations (first 30% of data). Blue = observed data, grey = held-out test, orange dashed = spatial model, purple dashed = game-based model.

5.4 Performance Metrics

We evaluate both models' predictions using *mean absolute error* (MAE) and *root mean squared error* (RMSE). Table 1 summarises the distribution of errors across all sites, including minimum, maximum, average, median, and quartiles.

Statistic	**Spatial Model**	**Game-Based Model**
MAE (units in cents, to three decimals)		
Min	1.959	0.052
Max	64.880	58.465
Mean	14.412	9.633
LQ	11.880	5.786
UQ	16.301	12.185
Median	14.181	9.106
RMSE (units in cents, to three decimals)		
Min	2.644	0.001
Max	2893.271	1713.006
Mean	173.586	119.918
LQ	110.410	67.940
UQ	204.465	145.181
Median	158.202	101.819

Table 1: Summary of predictive errors for the Spatial Model and the Game-Based Model. MAE = mean absolute error, RMSE = root mean squared error.

5.5 Discussion

Although both methods leverage spatial information from nearby stations, the simpler averaging model does not attempt to mimic large resets or strong undercutting. As a result, it tends to underestimate the steep price peaks and troughs—hence the higher average errors and extreme maxima (RMSE up to 2893). By contrast, the *game-based model* incorporates explicit jumps, yielding a better fit to cyclical extremes, although it remains imperfect.

Overall, these preliminary results confirm that spatial information is essential, but adding an Edgeworth-style reset mechanism improves predictive accuracy. Future refinements might include refining the threshold values (e.g. "30 cents above local maxima"), adjusting the randomness ε, or combining a more sophisticated spatial weighting with cyclical logic.

6 Conclusion and Future Extensions

6.1 Summary of Findings

This research set out to examine short-term retail fuel pricing cycles within an Australian context, using residual-based methods to isolate local price war dynamics from broader economic trends. After confirming the strong spatial dependency (via Moran's I) and cyclical behaviour (Maskin–Tirole model), we tested two approximate predictive models:

- **Spatial Model:** A nearest-neighbour averaging rule that updates each site's price based on the 10 geographically closest stations.
- **Game-Based Model:** A simplified *Edgeworth-like* scheme that allows for higher price "resets" and subsequent undercutting, referencing local maxima and averages.

Our results (Table 1) suggest that while purely spatial averaging already leverages the observed correlation among neighbouring sites, introducing game-like reset mechanics yields better predictive accuracy overall (e.g. lower mean absolute error). This underscores the importance of explicitly modelling cyclical price jumps and drops.

6.2 Potential Directions for Expansion

Although the proposed methods offer insight into local station interactions, several avenues remain open for further development.

6.2.1 Road-Network Adjacency and Graph Neural Networks

One immediate refinement is to leverage a road-based adjacency matrix (potentially derived from a service such as Google Maps or OpenStreetMap). Rather than simply taking the n closest sites by *geodesic* distance, one could define adjacency via actual driving distance or connectivity on major roads. This more realistic spatial graph could be used with advanced tools—*e.g.*, a *Spatial Graph Neural Network (SGNN)*—to propagate pricing signals along mapped routes. Such an approach might capture competition patterns more accurately, especially in urban areas with dense road networks.

6.2.2 A Dirichlet–Multinomial Transition Model

Another possible direction is constructing a *fully empirical* transition scheme, letting the data itself dictate the probability of moving from a given price (and local average) to another. In broad terms, one can:

- Discretise each site's price p and its local mean $\bar{p}$ into categories, forming a grid of possible states $(p, \bar{p})$.
- For each "row" (specific $(p, \bar{p})$ combination), observe the distribution of *next-step* prices in the dataset.

- Place a Dirichlet prior on the row's probability vector, and treat the subsequent occurrences as counts in a multinomial likelihood.

Because the Dirichlet and multinomial distributions are conjugate, one easily obtains a closed-form posterior for each row's transition probabilities. This setup (sometimes called a *row-wise Dirichlet–Multinomial* model) accommodates "smoothing" by choosing larger prior counts (add-λ or Lidstone smoothing). Once estimated, these transition probabilities can be employed in simulation to forecast likely price evolutions at each site, or even used to infer hidden states in a Markov chain.

6.2.3 PDE Models and Wave Propagation

With sufficiently granular data and adjacency information, an intriguing future project would be to formulate *partial differential equation (PDE)* models. In such PDE analogies, local price changes spread "outward" like waves, capturing the ripple effect of undercutting and resets. Such a continuous-space or continuous-time framework could theoretically represent how shocks in one region travel through neighbouring stations, eventually relaxing into cyclical equilibria reminiscent of Edgeworth cycles.

6.3 Concluding Remarks

The present work demonstrates that even relatively simple models—*spatial averaging* versus *game-based resets*—already differentiate well between stations' pricing patterns. By combining advanced spatial techniques (road-network adjacency, wave-based PDE interpretations) and richer data-driven transition models, future research can provide deeper insight into the microdynamics of retail fuel markets and help regulators and policymakers anticipate sudden price surges or identify tacit collusion practices.

Appendices

Summary of Previous Research

Key Studies in Fuel Price Dynamics

Below is the summary table detailing prior research contributions. Re-labelling it as Table 2 to avoid conflict with main text:

Author(s)	**Year**	**Study Focus**	**Key Findings**
Valadkhani [Val10]	2010	Long- and short-term effects on fuel prices in capital cities	Macroeconomic variables drive long-run price patterns; evidence of asymmetric price adjustments points to market inefficiencies.
Wang [Wan08]	2008	Collusive communication and price coordination in gasoline markets	Retail fuel price cycles mirror the Maskin-Tirole dynamic oligopoly model; strong evidence of coordinated pricing.
Bergantino et al. [BCI20]	2020	Spatial interaction and competition in retail fuel pricing	Marked spatial dependence in fuel prices; local competition plays a significant role in price formation.
Hogg et al. [Hog+12]	2012	The effect of branding on retail fuel prices	Branded stations often charge higher prices; independents foster greater competition and lower local prices.

Table 2: (Appendix) Summary of Previous Studies on Fuel Price Dynamics

Algorithm for Identifying Ridges

Algorithm 1 Wavelet-based Ridge Identification for Peak Detection (Full Pseudocode)

Require: CWT coefficient matrix C of size $N \times M$, ...

```
Initialise: Identify local maxima in row n = N ...
for each scale n = N − 1 down to 1 do
  for each existing ridge line r do
    Search for the nearest maximum ...
    if found one then
      Append the point to r ...
    else
      gap(r) = gap(r) + 1.
    end if
  end for
  Remove ridge lines whose gap > threshold.
  Create new ridge lines for any maxima ...
end for
Output: A collection of ridge lines across scales.
```

Haversine Distance Details

Here we provide the precise formula for the Haversine distance, used to define the spatial weights w_{ij} in Moran's I:

$$d_{ij} = 2R \arcsin\left(\sqrt{\sin^2\left(\frac{\phi_j - \phi_i}{2}\right) + \cos(\phi_i)\cos(\phi_j)\sin^2\left(\frac{\lambda_j - \lambda_i}{2}\right)}\right),$$

where R is Earth's radius, ϕ is latitude, and λ is longitude. For our application, we convert d_{ij} into a weight w_{ij} by a specified function (e.g. inverse distance or a fixed neighbourhood cutoff). More sophisticated spatial weighting methods could be employed, but Haversine suffices for this study.

Stationarity Checks (ADF)

To confirm stationarity of the residual series, we perform the Augmented Dickey-Fuller (ADF) test [DF79], checking whether the unit-root hypothesis can be rejected. For a series r_t,

$$\Delta r_t = \alpha + \beta t + \gamma r_{t-1} + \sum_{i=1}^{p} \delta_i \Delta r_{t-i} + \varepsilon_t.$$

Rejecting the null that $\gamma = 0$ indicates stationarity. Across all sites, p-values were sufficiently small to confirm stationarity in the residual pricing series, consistent with the broader literature on cyclical fuel pricing.

Entire Simulation Domain

A full simulation over the entire domain (excluding initial values)

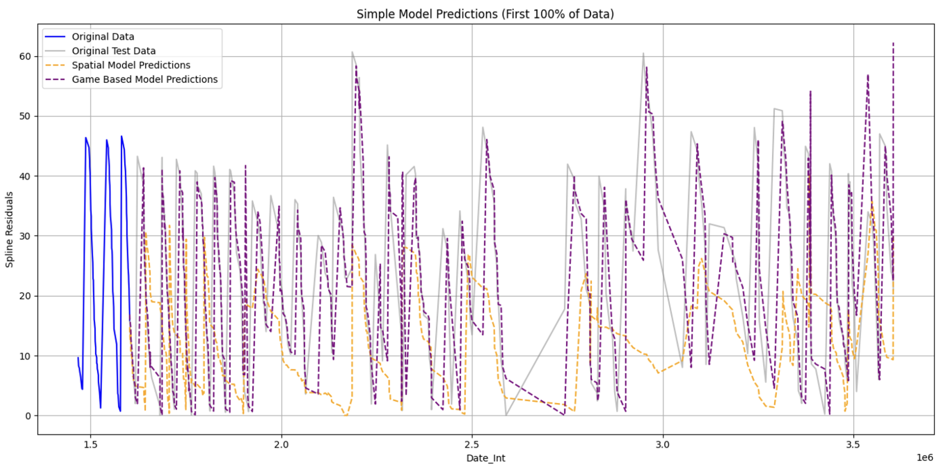

Figure 10: Forward simulation of the two models

References

[Mor50] P. A. P. Moran. "Notes on Continuous Stochastic Phenomena". In: *Biometrika* 37.1/2 (1950), pp. 17–23. (Visited on 02/13/2025).

[SG64] Abraham. Savitzky and M. J. E. Golay. "Smoothing and Differentiation of Data by Simplified Least Squares Procedures." In: *Analytical Chemistry* 36.8 (July 1964). Publisher: American Chemical Society, pp. 1627–1639. ISSN: 0003-2700. DOI: 10.1021/ac60214a047. URL: https://doi.org/10.1021/ac60214a047.

[DF79] David A. Dickey and Wayne A. Fuller. "Distribution of the Estimators for Autoregressive Time Series With a Unit Root". In: *Journal of the American Statistical Association* 74.366 (1979), pp. 427–431. (Visited on 02/13/2025).

[MT88] Eric Maskin and Jean Tirole. "A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles". In: *Econometrica* 56.3 (1988), pp. 571–599. (Visited on 02/13/2025).

[Bat93] Bruce J Bates. *Ten lectures on wavelets.* 1993.

[DKL06] Pan Du, Warren A. Kibbe, and Simon M. Lin. "Improved peak detection in mass spectrum by incorporating continuous wavelet transform-based pattern matching". In: *Bioinformatics* 22.17 (Sept. 2006), pp. 2059–2065. ISSN: 1367-4803. DOI: 10.1093/bioinformatics/btl355. URL: https://doi.org/10.1093/bioinformatics/btl355 (visited on 02/15/2025).

[Wan08] Zhongmin Wang. "Collusive Communication and Pricing Coordination in a Retail Gasoline Market". English. In: *Review of Industrial Organization* 32.1 (Feb. 2008). Copyright - Springer Science+Business Media, LLC. 2008; Last updated - 2024-12-06, pp. 35–52. URL: https://www.proquest.com/scholarly-journals/collusive-communication-pricing-coordination/docview/209917996/se-2.

[Val10] Abbas Valadkhani. "Modelling the Price of Unleaded Petrol in Australia's Capital Cities". In: *Australasian Accounting Business and Finance Journal* 4 (June 2010).

[Hog+12] Stephen Hogg et al. "A Spatial Econometric Analysis of the Effect of Vertical Restraints and Branding on Retail Gasoline Pricing". In: (Jan. 2012).

[KA16] Kangni R Kpodar and Chadi Abdallah. "Dynamic Fuel Price Pass-Through: Evidence from a New Global Retail Fuel Price Database". In: 2016.254 (2016). DOI: 10.5089/9781475563221.001. URL: https://www.elibrary.imf.org/view/journals/001/2016/254/001.2016.issue-254-en.xml.

[BCI20] Angela S. Bergantino, Claudia Capozza, and Mario Intini. "Empirical investigation of retail fuel pricing: The impact of spatial interaction, competition and territorial factors". In: *Energy Economics* 90 (2020), p. 104876. ISSN: 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2020.104876. URL: https://www.sciencedirect.com/science/article/pii/S0140988320302164.

[Com25] Australian Competition and Consumer Commission. *Petrol price cycles in major cities*. en. Text. Archive Location: Australia Last Modified: 2025-02-12T11:41:40+11:00 Publisher: Australian Competition and Consumer Commission. Feb. 2025. URL: https://www.accc.gov.au/consumers/petrol-and-fuel/petrol-price-cycles-in-major-cities (visited on 02/14/2025).

[Gov25] WA Government. *FuelWatch*. Government. 2025. URL: https://www.fuelwatch.wa.gov.au/retail/historic (visited on 02/15/2025).

[NSW25] Fair Trading NSW. *FuelCheck*. en. Last Modified: 2025-01-12T20:40:52.992491. 2025. URL: https://data.nsw.gov.au/data/dataset/fuel-check (visited on 02/15/2025).

[Sci] SciPy. *find_peaks_cwt*. URL: https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.find_peaks_cwt.html.

[Tre] Queensland Treasury. *QLD Fuel price reporting*. en. Last Modified: 2024-10-24T11:13:08+10:00. URL: https://www.energyandclimate.qld.gov.au/energy/vehicles-and-energy/fuel-price-reporting (visited on 02/15/2025).