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Climate Sensitivity Modelling

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Abstract

With the rapid growth in human population and industrialization, climate change has become a major global threat to the world. Previous studies have analyzed data on the temperature and rainfall to obtain trends in climate. It is believed that the increasing concentrations of carbon dioxide (CO_2) in the atmosphere contribute to the escalation of temperature [1] . Scientifically, CO_2 in atmosphere absorbs energy from the earth that would otherwise be re-emitted back to space and so traps heat [2] . Therefore, increasing CO_2 is believed to cause global warming.

Climate sensitivity, a term to characterize the response of the climate system to an imposed forcing, is arguably one of the most important quantities when discussing climate change. In technical terms, it is commonly defined as the equilibrium global mean surface temperature change that occurs in response to doubling of atmospheric CO_2 concentration [3] . This project aims to model climate sensitivity using a simple model with coupled partial differential equations for CO_2 concentration and temperature. Model results will be compared with available data.

1 Introduction

1.1 Global Surface Temperature Change Is Affected By The CO_2 Concentration

Due to the rapid growth in human activity and industrialization, primarily the burning of fossil fuels such as coal and oil after the Industrial Revolution in the 1800s, the carbon dioxide concentration in the atmosphere has increased (fig.1) .

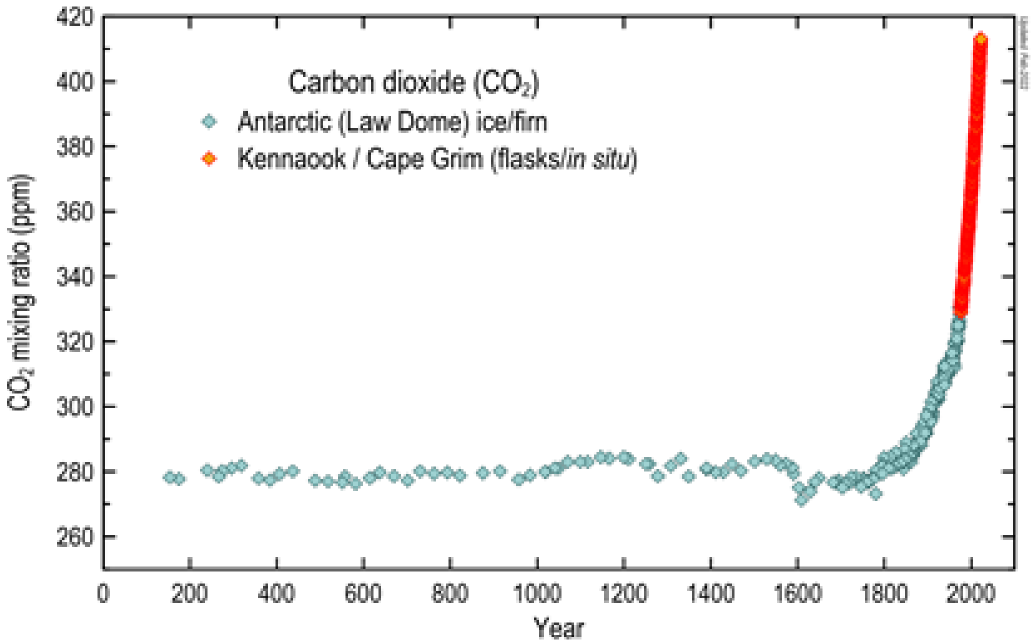

Figure 1: Atmospheric CO_2 concentrations (in ppm) Over The Last 2000 Years From CSIRO [4] .

As we all know, carbon dioxide (CO_2) is a greenhouse gas. This means that it causes an effect like a blanket in a greenhouse, trapping heat and warming up the interior. This effect is significant because, without the CO_2 that occurs naturally in the atmosphere, Earth may be too cold for life to survive there. However, the atmosphere is very sensitive to changing CO_2 concentration. Even though this gas makes up less than 0.1% of the atmosphere, it still can have a huge effect on how much heat the earth's surface retains [1] . The figure 2 shows that when energy from the sun reaches the top of our atmosphere, most of it passes through to Earth's surface, where it is absorbed. A portion of this energy is re-emitted, heading back towards space. At this stage, it interacts with molecules of CO_2 in a way that prevents some of it from escaping Earth's atmosphere [1] . The trapped heat energy leads to increased mean global surface temperatures [1] . Thus, the increasing CO_2 is believed to cause global warming. In this paper, I would like to use a mathematical model to explore how the global surface temperature change is affected by the CO_2 concentration.

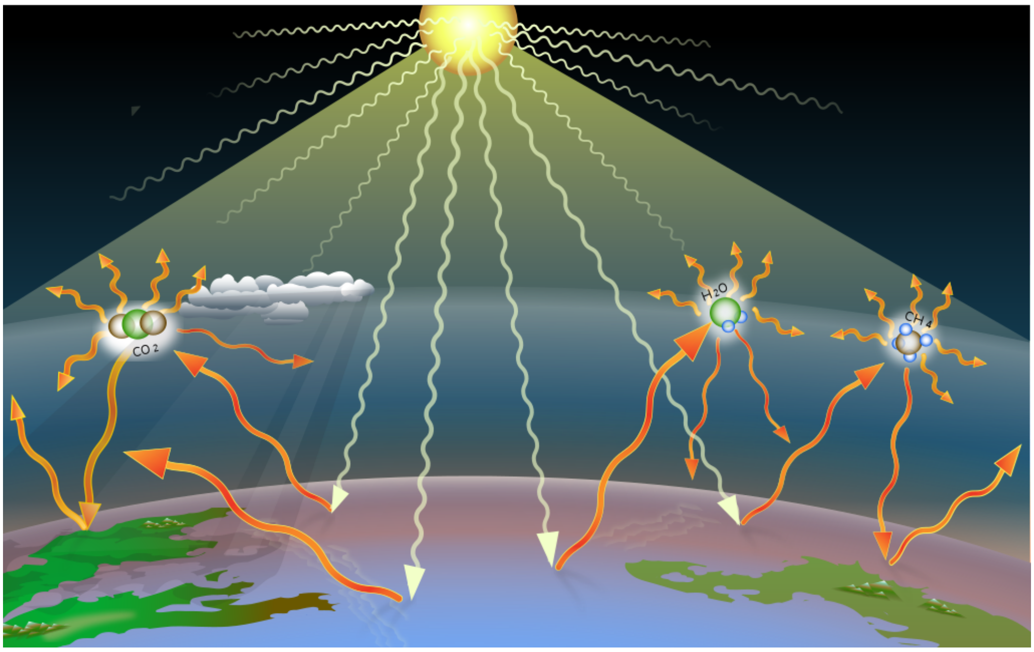

Figure 2: Simplified diagram showing how Earth transforms sunlight into infrared energy. Greenhouse gases like carbon dioxide and methane absorb the infrared energy, re-emitting some of it back toward Earth and some of it out into space [2].

1.2 Radiative Forcing

According to the Intergovernmental Panel on Climate Change (IPCC), Radiative forcing is a measurement of how much of a certain climatic component affects the amount of radiant energy that impinges on the surface of the Earth [7] . Climatic factors are divided between those caused primarily by human activity (such as greenhouse gas emissions and aerosol emissions) and those that are a result of natural causes (such as solar irradiance) [7] . The so-called forcing values are calculated for the period from 1750 to the present for each factor. Climate variables that influence the warming of Earth's surface are referred to as "positive forcing," whereas variables that influence the cooling of Earth's surface are referred to as "negative forcing." [7] . Radiative forcing on Earth is meaningfully evaluated at the tropopause and at the top of the stratosphere. It is measured in watts per square meter and is frequently expressed as an average over the entire surface area of the globe [7].

By the equation (1) , one can use radiative forcing to predict a subsequent change in steady-state (commonly referred to as "equilibrium") surface temperature resulting from that forcing [8] .

$$\Delta T_s = \lambda \Delta F[8] \tag{1}$$

where λ is commonly denoted the climate sensitivity parameter, usually with units $K/(W/m^2)$, and ΔF is the radiative forcing in W/m^2 [8] .

2 Data

2.1 Cape Grim Baseline Air Pollution Station

Cape Grim, on Tasmania's west coast, is one of the three premier Baseline Air Pollution Stations in the WMO-GAW (World Meteorological Organization's Global Atmosphere Watch program) network [4] .

Why the Cape Grim station is considered the baseline station? The clean air or 'baseline' sector for Kennaook / Cape Grim is depicted on the map below (fig.3) . Air masses from between 190 and 280 degrees that arrive at the Kennaook/ Cape Grim station have often travelled thousands of kilometers across the Southern Ocean [4] . Because this air is very well mixed and free of recent human and terrestrial impacts, meaning it represents the background or 'baseline' atmospheric composition for the mid-latitudes of the Southern Hemisphere [4] . The primary causes of climate change are the long-term changes in the composition of the baseline atmosphere.

The Kennaook / Cape Grim program originated from a commitment by the Australian Government to the United Nations Environment Program in the early 1970s to monitor and study global atmospheric composition for climate change purposes. As a result, the Kennaook / Cape Grim Baseline Air Pollution Station (KCG BAPS) first began measuring the composition of the atmosphere in April 1976 and has been in continuous operation since that date [6].

Kennaook / Cape Grim is a joint responsibility of the Bureau of Meteorology (BoM) and the Commonwealth Scientific and Industrial Research Organisation (CSIRO). All data are made available to interested parties, including Australian government agencies, industry, the public and international agencies [6].

Kennaook / Cape Grim data are more valuable than ever in understanding how our atmosphere is changing, as we track progress towards the goals of the Paris Agreement [6] .

2.2 Mauna Loa Baseline Air Pollution Station

We just said that Cape Grim data is the representative of the Southern Hemisphere. And how about the north Hemisphere? The map (fid.4) shows us the WMO-GAW network. We can see that Cape Grim and the Mauna Loa are the premier Global Baseline Station (fig.4) .

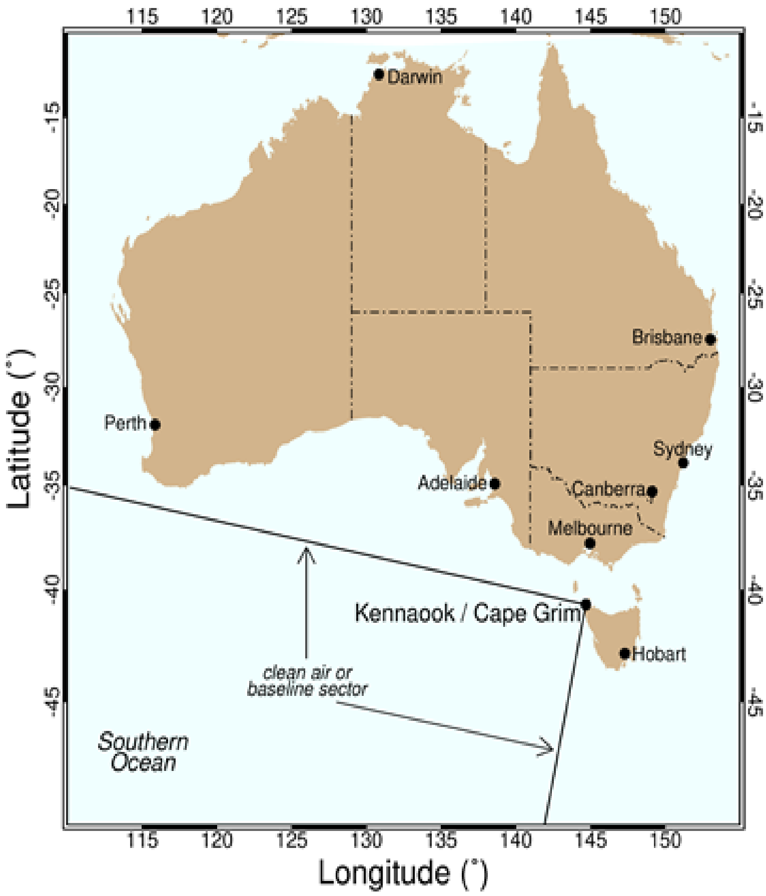

Figure 3: The Kennaook / Cape Grim station is positioned just south of the isolated north-west tip (Woolnorth Point) of Tasmania [6] .

In the Northern Hemisphere, the Mauna Loa Observatory (MLO) is situated at an altitude of 3397 metres, or 11,135 feet above sea level, on the north flank of the Mauna Loa Volcano on the Big Island of Hawaii [9] . The observatory, a world-class centre for atmospheric research, has been regularly observing and gathering information about atmospheric change since the 1950s [9] .

2.3 The Data of Cape Grim and Mauna Loa

To my surprise, Mauna Loa's data (fig.6) is almost similar to that of Cape Grim (fig.5) . They have almost the same slope. However, I had some difficulty in finding the temperature data in Mauna Loa. Then I choose to take Cape Grim's CO_2 concentration and temperature data as my data.

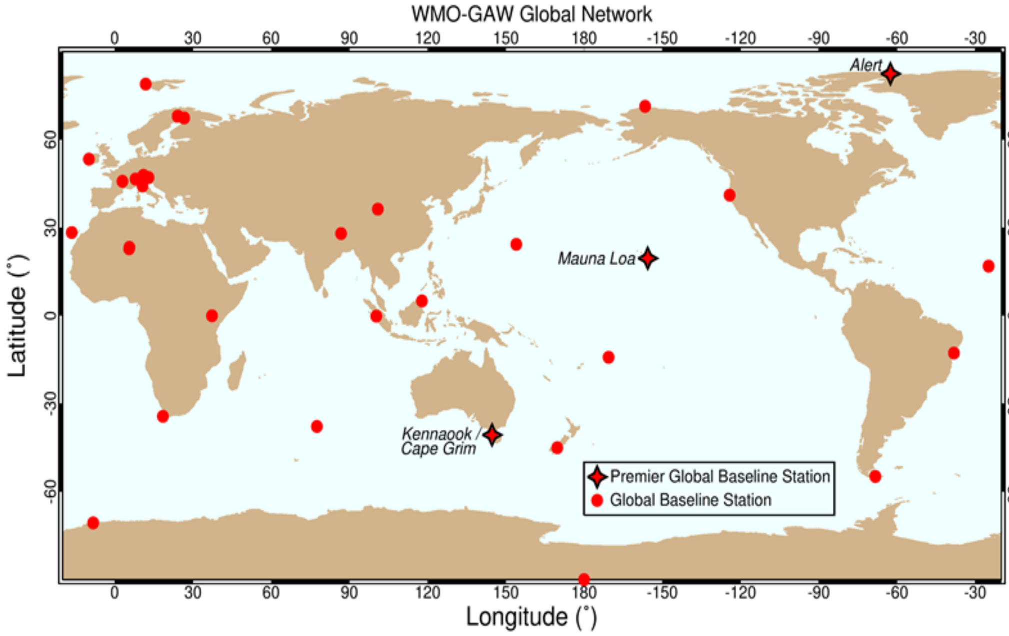

Figure 4: WMO-GAW Global Network [6]

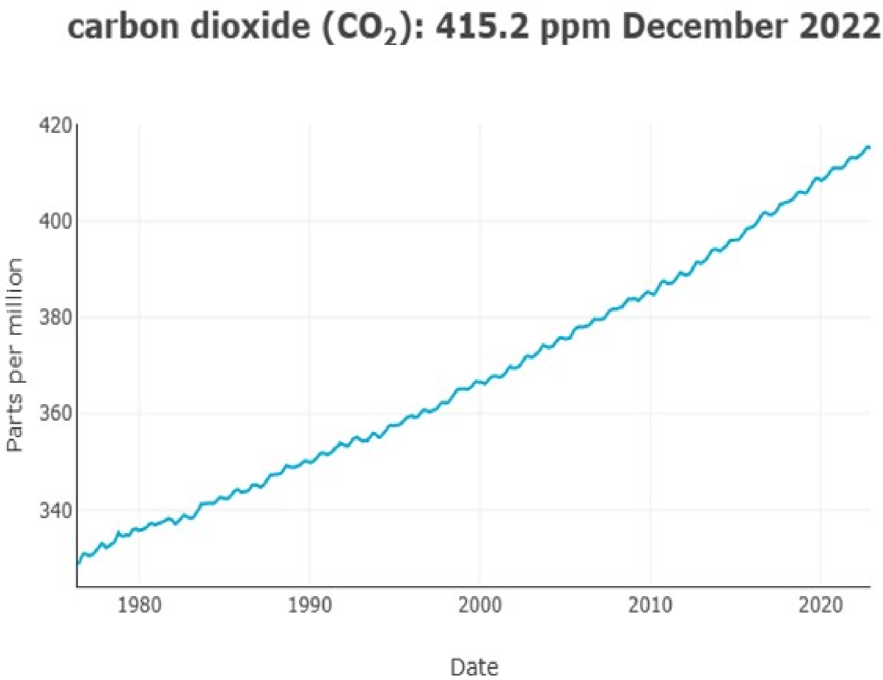

Figure 5: Cape Grim CO_2 Concentration Data from CSIRO [6]

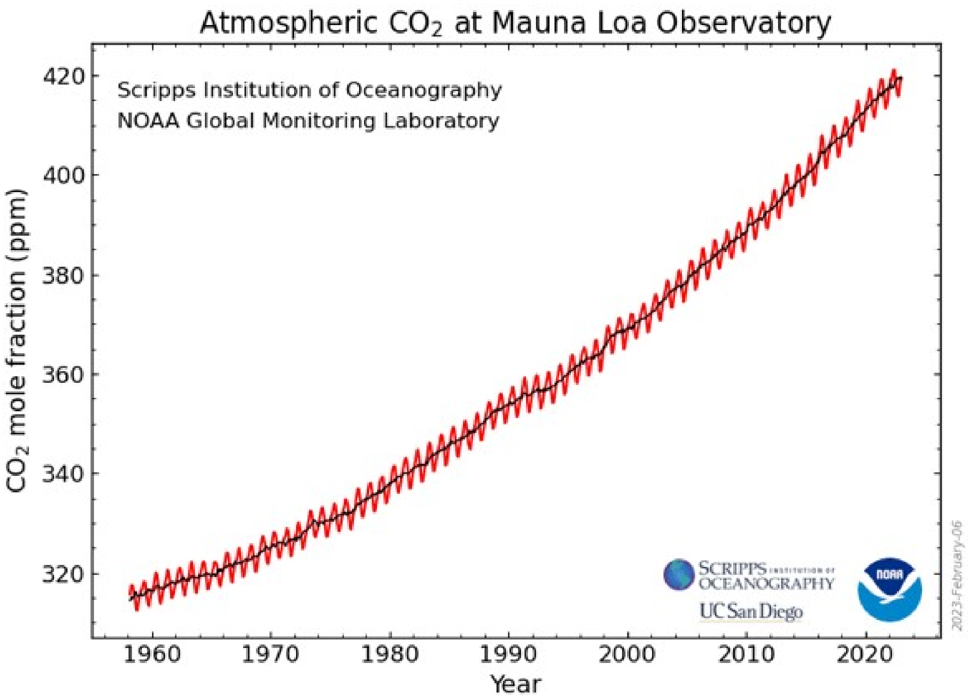

Figure 6: Mauna Loa CO_2 Concentration Data from NOAA [9]

3 Model

3.1 Global CO_2 Concentration And Globe Temperature Data

In the first step, I considered the global mean temperature and the global carbon dioxide concentration.

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(vC) = \frac{\partial}{\partial x}(D(C)\frac{\partial C}{\partial x}) + M. \tag{2}$$

where C is the CO_2 concentrations, and t is the year, and x is the position, and M is the rate of the production of CO_2, and D is the concentration diffusivity and v is the velocity of CO_2.

One might expect the time-dependent advection-diffusion equation to be applicable for modelling CO_2 concentration and temperature which varies with position and time (Equation.2) .

Additionally, human activities have led to increases in the concentrations of "well-mixed" CO_2 that are relatively evenly distributed because their molecules remain in the atmosphere for at least several years on average.

However, scientists can't measure these gases just anywhere, though. Consider the concentration of CO_2 in our atmosphere, which is one of the most important statistics for understanding climate change. Carbon dioxide is "well-mixed" into the atmosphere, meaning its concentration is more or less consistent around the world [6] . However, measuring too close to any major source of CO_2, such as a forest fire, a factory, or the gasoline-burning cars driving around a city, would skew the data. For this reason, scientists continuously measure the average level of CO_2 in the atmosphere by sampling the air at several remote sites around the world, including Mauna Loa and Cape Grim [6] .

Since we assume that CO_2 is well-mixed all around the global, so the global CO_2 does not depend on the positions or the locations. Therefore, I derive a ODE model like the one I show below.

$$\frac{dC}{dt} = M(t), \tag{3}$$

$$\frac{dU}{dt} = A\Delta F(C), \tag{4}$$

$$Initial\ Conditions. \tag{5}$$

where C is the CO_2 concentrations and U is the mean temperature.

Observing from the graph (fig.7) ,the Cape Grim data line is close to the linear function. Thus, I set M(t) as a constant is equal to 1.8457 ppm/year:

$$M(t) \approx constant = 1.8457\frac{ppm}{year}. \tag{6}$$

As I mentioned before, due to the radiative forcing equation:

$$\Delta T_s = \lambda \Delta F. \tag{7}$$

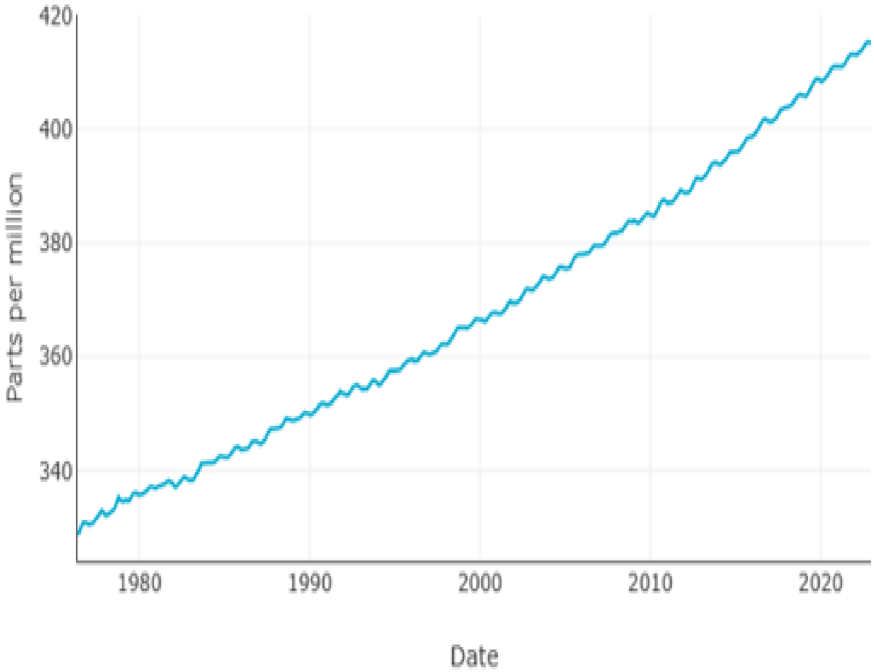

Figure 7: Cape Grim Data [6]

Therefore, I set the global mean temperature function like this:

$$\frac{dU}{dt} = A\Delta F(C). \tag{8}$$

Radiative transfer codes that analyse each spectral line for atmospheric circumstances can be used to determine the forcing ΔF as a function of a change in concentration for a well-mixed glasshouse gas. These calculations could be condensed into a formula in algebra that is unique to that gas [7] . Additionally, a simplified first-order approximation expression for carbon dioxide (CO_2) is [8] :

$$\Delta F = 5.35 \times ln\frac{(C_0 + \Delta C)}{C_0}. \tag{9}$$

where C_0 is a reference concentration in parts per million (ppm) by volume and ΔC is the concentration change in ppm. For the purpose of some studies (e.g. climate sensitivity), C_0 is taken as the concentration prior to substantial anthropogenic changes and has a value of 278 ppm as estimated for the year 1750 [8] .

Then, we can have :

$$\Delta F(C) = B \times log(\frac{C}{C_0}), \tag{10}$$

$$B = 5.35Wm^{-2}. \tag{11}$$

The next step is to make the variables dimensionless.

$$C = C_0\hat{C}, \tag{12}$$
$$U = \theta\hat{u}, \tag{13}$$
$$t = \frac{C_0}{M}\hat{t}. \tag{14}$$

Then,we can have the dimensionless equations:

$$\frac{d\hat{C}}{d\hat{t}} = 1, \tag{15}$$
$$\frac{d\hat{u}}{d\hat{t}} = \alpha log(\hat{C}), \tag{16}$$
$$\alpha = \frac{C_0AB}{\theta M}. \tag{17}$$

Moreover, because the data from Cape Grim starts in 1976, t=0 corresponds to the year 1976. Additionally, the mean temperature in 1976 is 14.7 and its carbon dioxide concentration is 328.861 ppm. The carbon dioxide concentration in 1750 is 278 ppm.

Thus, the initial conditions are:

$$\hat{t} = 0 \; in \; t = 1976, \tag{18}$$
$$\hat{u}(0) = 1(\theta = 14.7°C), \tag{19}$$
$$\hat{C}(0) = \frac{C(0)}{C_0} = 1.1830. \tag{20}$$

where $C_0 = 278$ ppm, $C(0) = 328.861$ ppm.

3.2 Solutions for Global Model

After these steps, we can obtain the solutions for global carbon dioxide concentration and temperature model.

The solutions are:

$$\hat{C}(\hat{t}) = \hat{t} + \hat{C}(0), \tag{21}$$
$$\hat{u}(\hat{t}) - \hat{u}(0) = \alpha[\hat{C}(\hat{t}) \; log(\hat{C}(\hat{t})) - \hat{C}(\hat{t})] - \alpha[\hat{C}(0) \; log(\hat{C}(0)) - \hat{C}(0)]. \tag{22}$$

However, we have yet to determine what the value of α is. We know that alpha is equal to the equation we mentioned earlier.

$$\alpha = \frac{C_0AB}{\Theta M}. \tag{23}$$

We already know the values of C_0, B, θ and M, but we need to know the value of A. Thus, I chose to use the best least squares method to obtain the best value for α (equation.8) .

We already got the solution of the temperature function like:

$$\hat{u}(\hat{t}) - \hat{u}(0) = \alpha[\hat{C}(\hat{t}) \; log(\hat{C}(\hat{t})) - \hat{C}(\hat{t})] - \alpha[\hat{C}(0) \; log(\hat{C}(0)) - \hat{C}(0)]. \tag{24}$$

Then, I make these terms as f(t), because they are function of C and C is the function of t, so they are also function of t, which is:

$$\alpha[\hat{C}(\hat{t})\ log(\hat{C}(\hat{t})) - \hat{C}(\hat{t})] - \alpha[\hat{C}(0)\ log(\hat{C}(0)) - \hat{C}(0)] = f(t), \tag{25}$$

$$\hat{u}(\hat{t}) - \hat{u}(0) = \alpha f(t). \tag{26}$$

Moreover,

$$\alpha = [\hat{u}(\hat{t}) - \hat{u}(0)](ActualData)\backslash f(t). \tag{27}$$

where $\hat{u}(\hat{t})$ and $f(t)$ are vectors of data and MATLAB notation is used. In addition, I used the temperature data in Cape Grim from 1976 to 2022, so $\hat{u}(\hat{t}) - \hat{u}(0)$ is the actual difference for every two years.

Finally, we can get the alpha is equal to 0.4671.

This graph (fig.8) shows the relationship between the temperature difference (du) and CO_2 concentrations.

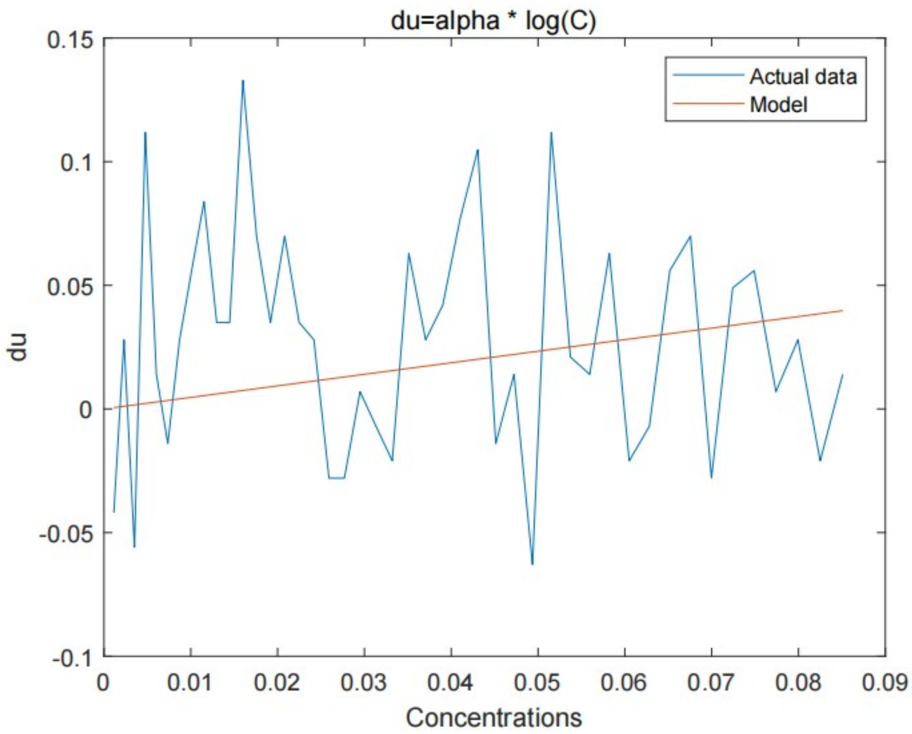

Figure 8: $du = \alpha * log(C)$

Most importantly, these are the solutions graphs (fig.9 & fig.10).

In the concentration graph (fig.9) , we can see that the actual data and the model line are quite similar. The actual data is from 1976 to 2022 in Cape Grim. Moreover, in the temperature graph (fig.10) , the actual

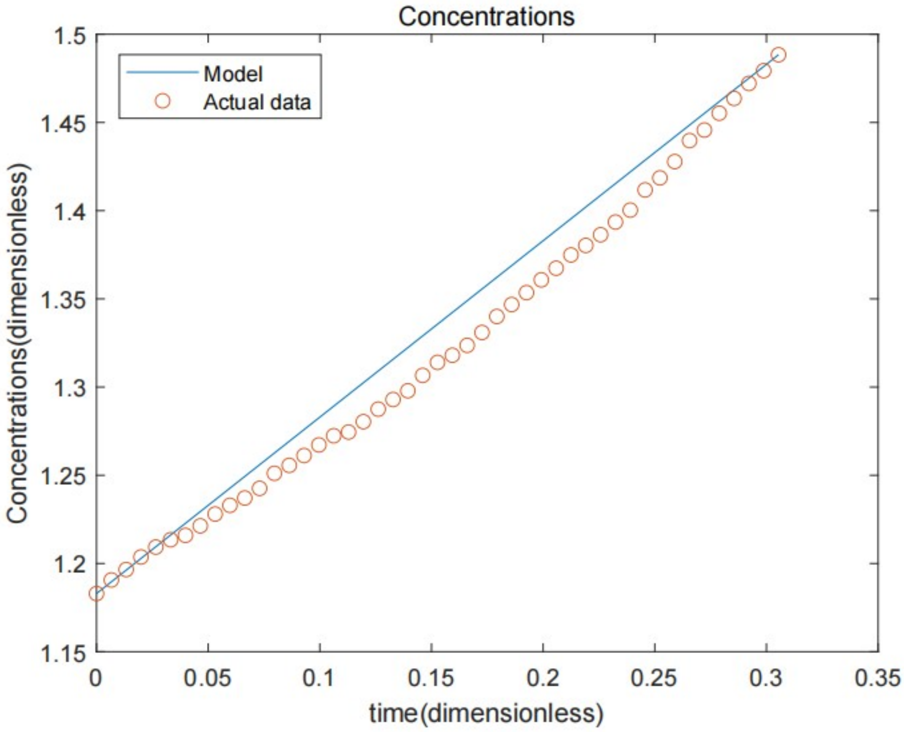

Figure 9: Global $C0_2$ Concentration Function

data which is from 1976 to 2022 in Cape Grim is quite spiky and our model line does not fit really well as the temperature is affected by a lot of factors and not just $C0_2$ concentrations.

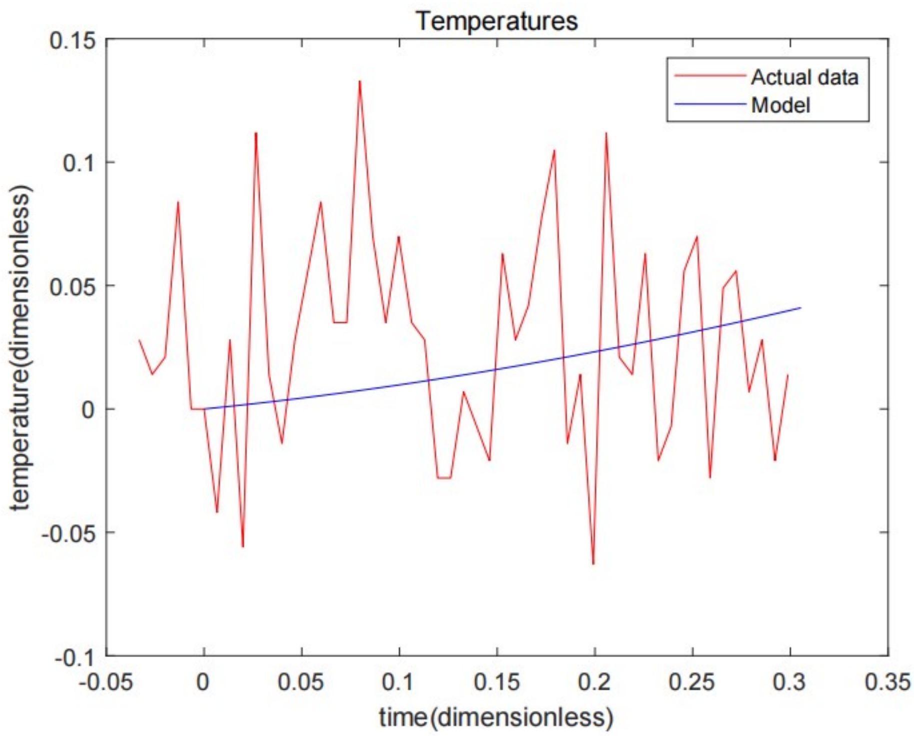

Figure 10: Global Mean Temperature Function

4 Climate Sensitivity

Then, we go back to model the climate sensitivity.

Climate sensitivity is one of the most important quantities when discussing climate change. The Intergovernmental Panel on Climate Change (IPCC) uses the term "climate sensitivity" to explain how changing glasshouse gas concentrations affect Earth's temperature. In particular, it is defined as the equilibrium global mean surface temperature change that occurs in response to doubling of atmospheric CO_2 concentration.

As a result, the climate sensitivity model is set like this:

- Concentration Equation

$$C = 2C_\tau. \tag{28}$$

- Climate Sensitivity Equation

$$du = \alpha C_\tau[2log(2C_\tau) - log(C_\tau) - 1]. \tag{29}$$

4.1 Solutions

The solution graph (fig.11) is:

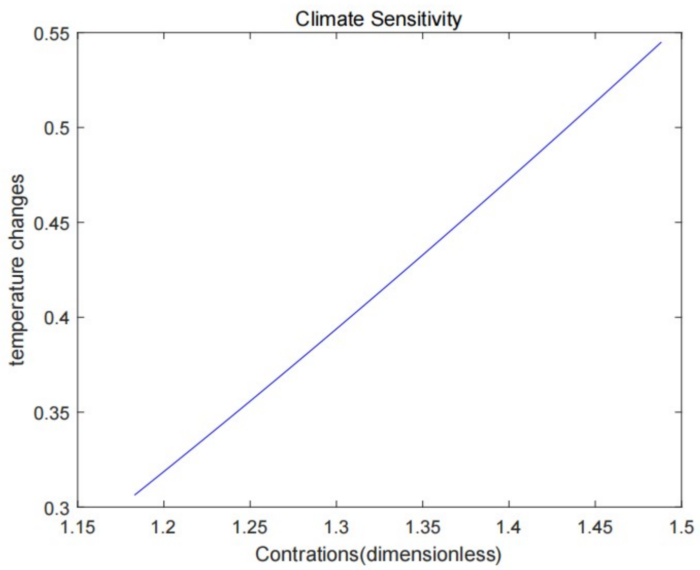

Figure 11: Climate Sensitivity

Surprisingly, although the climate sensitivity function is logarithmic, the images show it is nearly linear.

5 Spatially Varying CO_2

Next, we next consider adding x, the position, into the equation, so we rewrite the model as a PDE model.

The generalised 1-D diffusion equation is:

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} + M(x,t). \tag{30}$$

However, we do not know the velocity (v) and the diffusivity (D) in each area. Thus, I decide to forget these two terms and then we can get the regional concentration equation like:

$$\frac{\partial C}{\partial t} = M(x,t). \tag{31}$$

Additionally, the term M is the rate of production of carbon dioxide concentration, per year per ppm. There must be places on our planet where the concentration of carbon dioxide is high, such as large cities, and there must be places where it is low, such as forests. Different places have different their carbon dioxide concentration data. As a result, I set M as $M(x,t)$ and $M(x,t)$ is the rate of regional carbon dioxide concentration production. If we can obtain the regional carbon dioxide concentration data, we will derive a PDE model:

$$\frac{\partial C}{\partial t} = M(x,t), \tag{32}$$

$$\frac{dU}{dt} = A\Delta F(C), \tag{33}$$

$$Initial\ Conditions. \tag{34}$$

However, I need help finding data on CO_2 concentrations in the area. Then, by looking at each region's emission and temperature data, I used the data from Cape Grim as a baseline. Then, adding the sine function would simulate the CO2 concentration in each region going up and down from the baseline.

Thus, I choose to use the same ODE model as before (Equation.(15) (16)), but assuming $C(0) = 1.1830 + Dsin(2\pi x)$ for $0 \leq x \leq 1$ and $D = 0.15$.

$$\hat{C}(\hat{x},\hat{t}) = \hat{t} + 1.1830 + Dsin(2\pi\hat{x}), \tag{35}$$

$$\hat{u}(\hat{x},\hat{t}) - \hat{u}(\hat{x},0) = \alpha[\hat{C}(\hat{x},\hat{t})\ log(\hat{C}(\hat{x},\hat{t})) - \hat{C}(\hat{x},\hat{t})] - \alpha[\hat{C}(\hat{x},0)\ log(\hat{C}\hat{x},0)) - \hat{C}(\hat{x},0)]. \tag{36}$$

5.1 Results

The solution graphs (fig.12 & fig.13) are:

In both graphs (fig.12 & fig.13) , the bottom line represents the situation in 1976, rising in order, so the top line represents the situation in 2022. From the temperature one (fig.13) , we can see that the temperature change can be negative where the CO_2 concentration is low and positive where the CO_2 concentration is high. The higher the CO_2 concentration, the greater the temperature change.

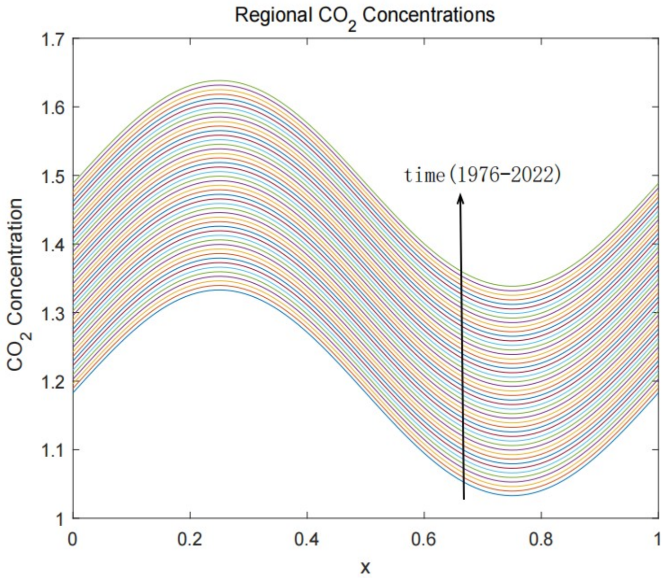

Figure 12: Regional CO_2 Concentrations

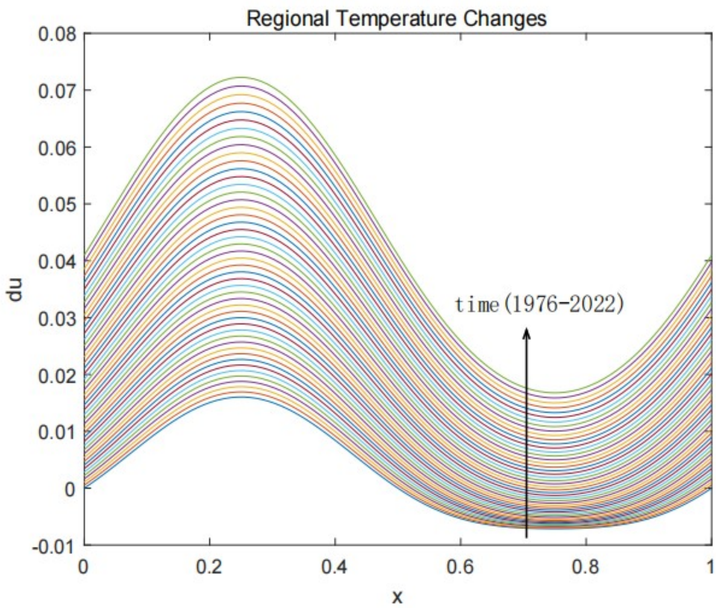

Figure 13: Regional Mean Temperature

6 Discussions And Conclusions

6.1 Observation results show that

- Compared with the atmospheric carbon dioxide concentration in 1750, the present carbon dioxide concentration has not doubled.
- Interestingly, we found that although the climate sensitivity equation is a log function, it is very close to a linear function.
- The temperature data in various places is affected by many factors, and it is a bit difficult to fit the temperature change by relying on the concentration of carbon dioxide alone.

6.2 Data Problem

- It is very difficult to find regional carbon dioxide concentration data, can only find regional CO2 emissions data.
- Most of the current data only include global carbon dioxide concentration data.

7 Acknowledgements

The author would once again like to acknowledge the supervisor Professor Yvonne Stokes, for her guidance in this research project and the help of University of Adelaide.

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