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Analytic Solution and Theory for the Size and Shape of Skyrmions as a Function of Magnetic Material Properties

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Abstract

The goal of this research project is to find an analytic solution for the size and shape of skyrmions as a function of magnetic material properties. Our analytic solution and theory are simpler and easier to be understood by those who are not from mathematical fields compared to the results developed by Büttner *et al.*. The expressions developed reveal the underlying physics of the problem. We use an analytic method which was developed from solving magnetic domain wall problems. We identify five major energy contributions that determine the size and shape of skyrmions: Dzyaloshiniskii-Moriya interaction (DMI) energy, exchange interaction energy, anisotropy energy, Zeeman energy and demagnetization energy. Piecewise functions were used to approximate the magnetization angle function to simplify the energy density expressions for each of the contributions. The energy per unit area was calculated for a thin film by integrating over cylindrical coordinates. In order to examine the solution accuracy, the results were plotted alongside plots using the function from Büttner *et al.* That function is more accurate than our results but is very complicated. Our energy contributions are generally close to that of Büttner *et al.*. We produced an analytical result to describe the size and width of skyrmions by minimizing the total energy of the skyrmion.

1 Introduction

Magnetic skyrmions are tiny swirls of local magnetization in magnetic materials. As depicted in Figure 1, the direction of magnetization of a typical skyrmion varies from down $-z$ direction in the centre region to up $+z$ direction at the edge of the skyrmion. This variation occurs along the radial axis ρ.[1] A skyrmion is described in some papers as a topologically protected quasiparticle, which has unique properties as a whole rather than its individual components. Its stability and dynamics depend strongly on its topological properties. [1] Skyrmions are promising magnetic structures in materials for transportation and storage of digital information. [2]

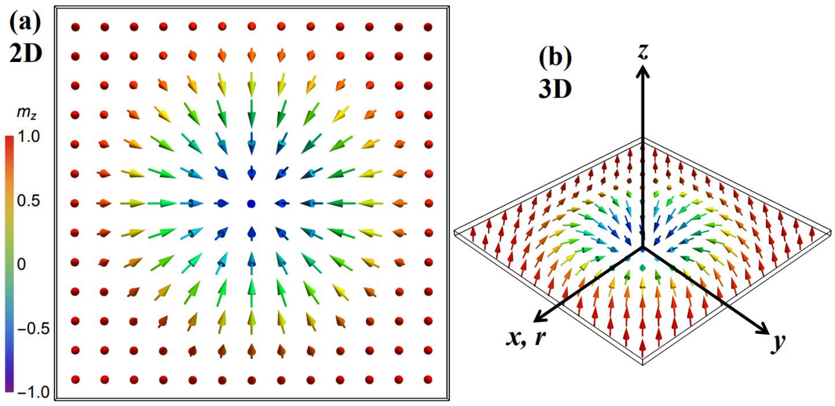

Figure 1: The schematic of a typical skyrmion: arrows show the direction of magnetization, and the colour indicates the projection along the out-of-plane z direction. The local magnetization gradually rotates from $-z$ direction (centre of the skyrmion) to $+z$ direction (edge of the skyrmion) along the axis ρ.(a) 2D view from above the skyrmion. (b) 3D view of the skyrmion.

There are three features of skyrmions, which make them one of the best candidate materials for next generation information storage technology. Due to skyrmions being a topologically protected atomic spin configuration, they are relatively stable compared to other magnetic structures such as magnetic vortices or bubbles.[4] Skyrmions can be extremely small, down to the "single-digit nanometre scale" with the support of the interfacial Dzyaloshinskii-Moriya interaction (DMI). DMI arises from spin-orbit coupling and stabilizes smaller skyrmion structures at room temperature. [2] Moreover, skyrmions can be created, deleted and moved like a single particle through a material using very small amounts of electric current. The current densities used in moving skyrmions (through magnetic material such thin films) can be several orders of magnitude smaller than the one used in moving magnetic domain walls. [2] Hence, the future digital data storage device by using magnetic skyrmions would have better stability, higher information density and much lower energy costs as well as ease of manipulation via electric current. [5] Although today's hard-disk drive can achieve high densities of information storage, they have very complex and fragile mechanical parts. On the other hand, skyrmions do not require extra parts to be moved, and skyrmions can achieve even higher bit density compared to today's magnetic data storage devices.[4]

Besides data storage devices, skyrmions are also a good candidate for some logic gates. [2] However, both future information and communication technologies require individual skyrmions to be small and stable at room temperature and in zero or very small applied fields.[1] In a sea of choices of potential material, a good mathematical model that can predict the size and shape and behaviour of a skyrmion as magnetic properties of the material are varied, would enable material scientists to search for the 'holy grail' of skyrmionic devices in bright daylight rather than in the dark.

Büttner *et al.* developed a very complicated analytical framework and numerical solutions to predict the property of isolated skyrmions in any magnetic thin film.[5] The downside of the work by Büttner *et al.* is that the analytical model is complicated for scientists who do not have a mathematical background. Moreover, some parts of the framework rely on numerical data fitting to a function rather than being purely derived from physics theories. Hence, the goal of our work is to develop simpler analytical theories that actually represent physical properties of skyrmions. The solution should be easier to use and understand by material scientists and engineers from different academic fields.

In this report, the aim is to present the analytic theories that we developed for describing the size and shape of skyrmions in thin films, as a function of magnetic material properties. In section II, our analytic energy densities contributions for skyrmions are described in detail. In Section III, energy minimization to find the skyrmion size is detailed. Finally, the conclusion and future work will be discussed in section IV.

Statement of Authorship: All works presented in this report are Ellen Lu's calculation except equation (17) and (18), which were proved by Karen Livesey. The results were confirmed and examined by Karen Livesey.

2 Skyrmion energy contributions

In the following subsections, the major energy density contributors are introduced and they are presented in cylindrical coordinates. The piecewise function for the magnetization inside a skyrmion is presented in detail, allowing analytic integration of the energy densities. Then, the final energy per unit area of each energy contributors are analysed in graphs.

2.1 Dzyaloshinskii-Moriya interaction energy

DMI is the antisymmetric exchange interaction, which was initially found in weak ferromagnetic materials. It arises from spin orbit coupling. [6, 7] It is the interaction that gives rise to the formation of skyrmions in magnets. $\vec{D}$ is the DMI vector with units of J/m^2 and it is assumed that D is positive. The expression for the DMI energy density in Cartesian coordinates [9] is

$$w_{DMI} = -D\left([\hat{y}\times\hat{z}]\cdot\left[\hat{m}\times\frac{\partial\hat{m}}{\partial\hat{y}}\right] + [\hat{x}\times\hat{z}]\cdot\left[\hat{m}\times\frac{\partial\hat{m}}{\partial\hat{x}}\right]\right), \tag{1}$$

where $\hat{m}$ is the magnetization vector, and $\hat{x}, \hat{y}, \hat{z}$ are unit vectors. Transforming the energy density equation to cylindrical coordinates is not only for the convenience of calculation but also a better representation of skyrmion structure, since it has cylindrical symmetry. By close investigation of Equation (1), we realise that the cross product of $\hat{y}$ and $\hat{z}$ is the unit vector $\hat{x}$. Similarly, $\hat{x}\times\hat{z} = -\hat{y}$. This means the the dot products between $\hat{x}$ and any other vectors has only the component in x direction left. Thus the equation can be written as:

$$w_{DMI} = D\left[\left(m_y\frac{\partial m_z}{\partial y} - m_z\frac{\partial m_y}{\partial y}\right) + \left(m_x\frac{\partial m_z}{\partial x} - m_z\frac{\partial m_x}{\partial x}\right)\right]. \tag{2}$$

Using the relationship between Cartesian and cylindrical coordinates, we can obtain the magnetization unit vector $\hat{m}$ components

$$\begin{aligned} m_x &= m_\rho\cos\phi - m_\phi\sin\phi \\ m_y &= m_\rho\sin\phi - m_\phi\cos\phi \\ m_z &= m_z. \end{aligned} \tag{3}$$

The chain rule can be used to obtain the coordinate transformation for the partial derivatives:

$$\begin{aligned}\frac{\partial}{\partial x} &= \frac{\partial \rho}{\partial x}\frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial x}\frac{\partial}{\partial \phi} + \frac{\partial z}{\partial x}\frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} &= \frac{\partial \rho}{\partial y}\frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial y}\frac{\partial}{\partial \phi} + \frac{\partial z}{\partial y}\frac{\partial}{\partial z}\end{aligned} \tag{4}$$

Substituting Equation (3) and Equation (4) into Equation (2), we can obtain a function that only have cylindrical components. As shown in Figure 1, there is no change of magnetization along the azimuthal axis ϕ. So, the energy density function can be further simplified into:

$$w_{DMI} = -D\left(m_\rho \frac{\partial m_z}{\partial \rho} - m_z \frac{\partial m_\rho}{\partial \rho} + \frac{m_z m_\rho}{\rho}\right). \tag{5}$$

2.2 Other energy contributions

The exchange interaction energy density in Cartesian coordinates is expressed as

$$w_{ex} = A\left\{\sum_{i=x,y,z}\left[\left(\frac{\partial m_i}{\partial x}\right)^2 + \left(\frac{\partial m_i}{\partial y}\right)^2 + \left(\frac{\partial m_i}{\partial z}\right)^2\right]\right\}. \tag{6}$$

Using the same technique as for the DMI energy calculation, we can transform the coordinates system of the equation above. Similarly, all the derivatives with respect to ϕ go to zero due to no change in the magnetization relative to azimuthal direction. The exchange energy density in cylindrical coordinates is given by:

$$w_{ex} = A\left[\left(\frac{\partial m_\rho}{\partial \rho}\right)^2 + \left(\frac{\partial m_\phi}{\partial \rho}\right)^2 + \left(\frac{\partial m_z}{\partial \rho}\right)^2 + \left(\frac{\partial m_\rho}{\partial z}\right)^2 + \left(\frac{\partial m_\phi}{\partial z}\right)^2 + \left(\frac{\partial m_\phi}{\partial z}\right)^2 + \frac{{m_\phi}^2 + {m_\rho}^2}{\rho^2}\right], \tag{7}$$

where A is the exchange stiffness constant for a material. Because the skyrmion exists in a thin film in the z direction, there is no change of magnetization on the z direction, all the derivatives with respect to z vanish in the Equation (7). The ϕ component of magnetization does not change along the ρ direction either. The term $\frac{\partial m_\phi}{\partial \rho}$ goes to zero. Now the Equation (7) can be simplified as below:

$$w_{ex} = A\left[\left(\frac{\partial m_\rho}{\partial \rho}\right)^2 + \left(\frac{\partial m_z}{\partial \rho}\right)^2 + \frac{{m_\phi}^2 + {m_\rho}^2}{\rho^2}\right] \tag{8}$$

Anisotropy energy describes the preference of the magnetization to point in or out of the plane, $\pm z$. Its energy density is given by:

$$w_{anis} = K(1 - {m_z}^2), \tag{9}$$

which is unchanged in cylindrical coordinates.

The Zeeman effect describes the lowering of the energy when the magnetization points along the applied field direction, taken to be $+z$ here. Its energy density can be expressed as:

$$w_{zee} = \mu_0 M_s B(1 - m_z) \tag{10}$$

where μ_0 is the permeability of free space with unit $[m \cdot kg \cdot s^{-2} \cdot A^{-2}]$, M_s is the saturation magnetization with unit $[A/m]$ and B is the magnetic induction with unit T.

2.3 Piecewise magnetization angle approximation

It would be too complicated to integrate over the original magnetization angle θ function ($\theta(\rho) = \pm 2\tan^{-1}\left[e^{(\rho-R)}/\Delta\right]$), where R is the location of the magnetization variation and Δ is the variation width. Thus, we need a simpler function to approximate the original θ, which would ultimately produce a reliable analytic result for the energy. Our skyrmion piecewise function is formulated as shown in Figure 2 below. We define R as size of the skyrmion and Δ

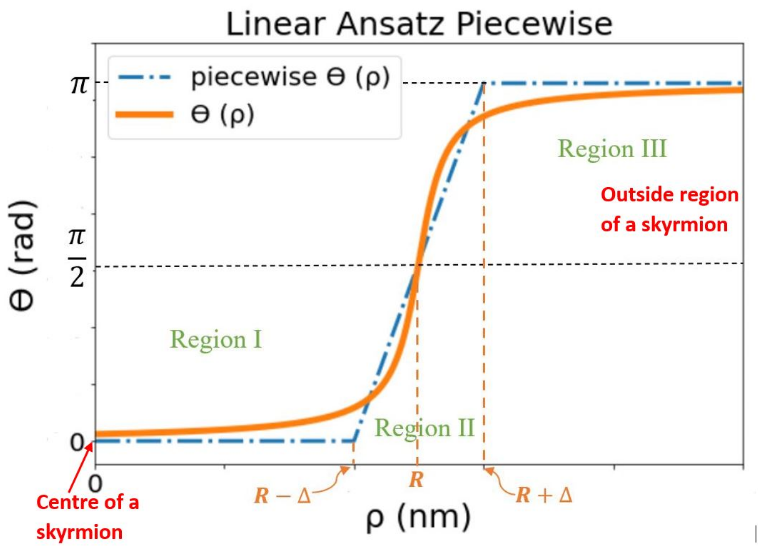

Figure 2: Piecewise function describing the Skyrmion magnetization angle profile: the Orange line presents the local magnetization angle θ function as function of radial direction ρ; The blue dash line presents the Linear Ansatz function of the angle θ.

as the width of skyrmion. The magnetization angle θ can be approximated by a linear ansatz function of ρ:

$$\theta(\rho) = \begin{cases} 0, & 0 < \rho < (R-\Delta), \quad \text{region I} \\ \frac{\pi}{2\Delta}\rho - \frac{\pi}{2\Delta}(R-\Delta), & (R-\Delta) < \rho < (R+\Delta), \quad \text{region II} \\ \pi, & \rho > (R+\Delta), \quad \text{region III} \end{cases} \tag{11}$$

As shown in Figure 2, the function describes the magnetization angle θ from the $+z$ direction from the centre of the skyrmion to the outside region. This allows the local magnetization function can be express as $m_z = \cos[\theta(\rho)]$ and $m_\rho = \sin[\theta(\rho)]$.

Substituting Equation (11) into cylindrical DMI energy density (Equation 5), one sees that regions I and III do not contribute to the total DMI energy. Hence, we only needed to integrate the energy density over region II, the DMI energy per unit area was obtained:

$$\int_0^\infty d\rho \int_0^{2\pi} d\phi \int_{-\infty}^{+\infty} dz \; (\rho \cdot w_{DMI}) \equiv E_{DMI} = 2\pi t \cdot \pi R D \tag{12}$$

where t is the thickness of skyrmion material in metres, through the z direction. The factor of 2π comes from integration over the azimuthal angle ϕ. The equation shows that the DMI energy increases with the skyrmion size R and thickness t of the material.

In order to examine the accuracy of our approximation functions, we plot our energy densities as a function of skyrmion size R and width Δ on the same graph that was created by using the results from Büttner *et al.*.[5] The results from Büttner *et al.* have high accuracy and are proven by comparison to experimental and simulation data. [5]

Figure 3 is the DMI energy density as a function of width Δ and size R. It shows that DMI has no impact on the width Δ of the skyrmions. However, DMI energy linearly decreases with increase of the size R. This might indicate that the higher DMI energy does make smaller skyrmions more stable. Our analytic approximation (solid lines) give a perfect fit with the result from the work of Büttner *et al.* (dashed lines) [10].

Substitution of Equation (11) into the exchange energy density given in Equation (8) leads to the energy by integrated over region II. Regions I and III do not contribute to the total energy. However, the integration can not be solved easily at first. The exchange energy integral is given by

$$E_{ex} = 2\pi t A \int_{R-\Delta}^{R+\Delta} d\rho \left\{ \rho \left(\frac{\pi}{2\Delta}\right)^2 + \frac{1}{\rho} \sin^2 \left[\frac{\pi}{2\Delta}(\rho - R + \Delta)\right] \right\}. \tag{13}$$

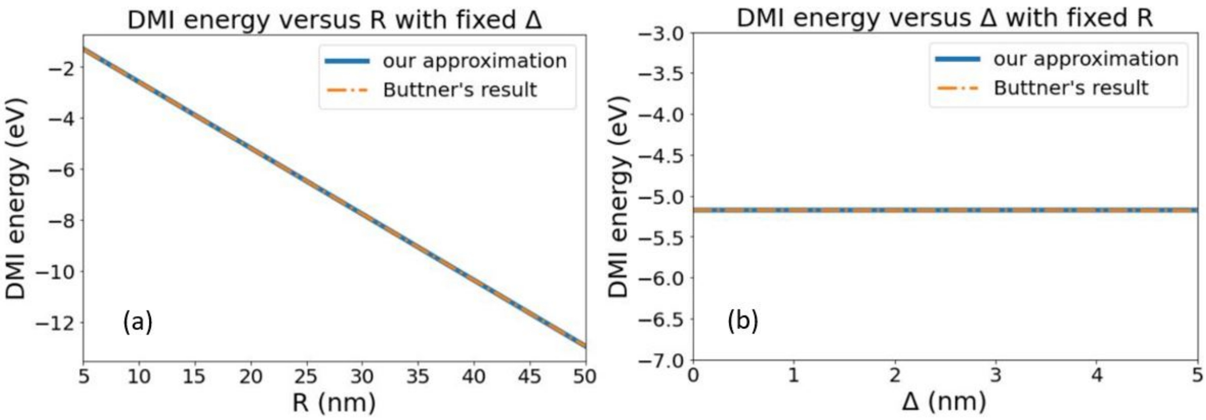

Figure 3: DMI energy per unit area as an function of Width Δ & size R: (a) with fixed width $\Delta = 2$nm, DMI energy linearly decreases with increase of the size R. Our approximation of DMI energy (blue line) is a perfect fit with the results from Büttner et al. [10] (orange line); (b) with fixed size R = 20nm, DMI has no effect on width of the skyrmions, and the result from Büttner's group also confirmed this.

A Taylor expansion to first order is used to approximate the value $\frac{1}{\rho}$ near $\rho \sim R$. This is where the integrand has its main contribution. One obtains $1/\rho \sim \left(\frac{2}{R} - \frac{\rho}{R^2}\right)$. This allows us to obtain a much simpler expression for the exchange energy, namely

$$E_{ex} = 2\pi t A \left(\frac{\pi^2 R}{4\Delta} + \frac{2\Delta}{R} \right). \tag{14}$$

Higher order approximations by using Taylor series were examined, the first order gave the closest and simplest result. The result is plotted along side the result from Büttner *et al.* in Figure 4 to examine the accuracy.

The Figure 4 is the exchange energy as a function of the size (R) and transition width (Δ) of the skyrmion. It shows that the Taylor expansion to first order preforms better than the second order approximation. Although, our function slightly underestimates the exchange contribution, it is sufficient for energy minimization (see the next Section).

Solving for the anisotropy and Zeeman energy contributions is easier by substituting the piecewise functions [equation (11)] into the energy density equations [(9) and Equation (10)] accordingly. The anisotropy energy is result is

$$E_{anis} = (2\pi t) \cdot 2\Delta R K, \tag{15}$$

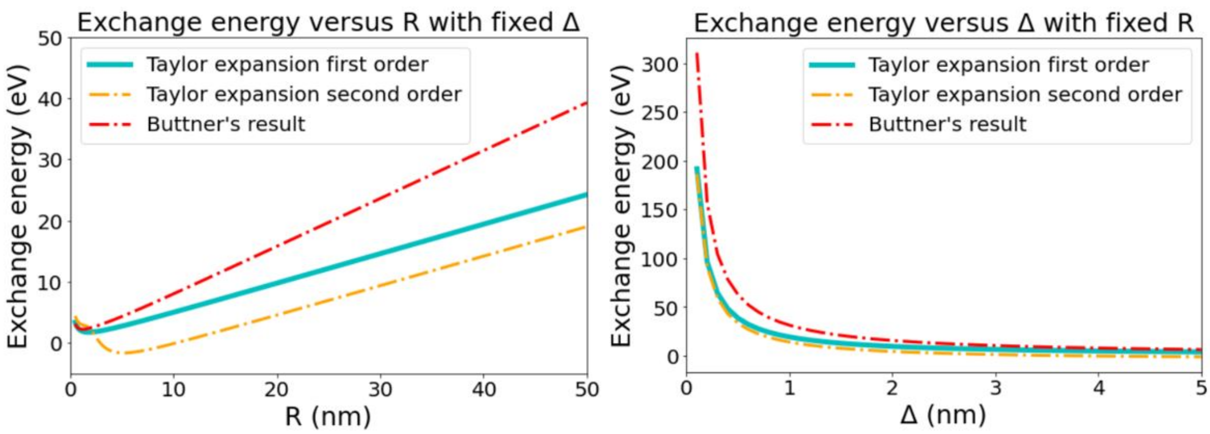

Figure 4: The exchange energy per unit area as a function of width and size of the skyrmion: (a) with fixed width Δ = 2nm, the exchange energy increase with size R; (b) with fixed size R = 20nm, the exchange energy exponentially decay with increase of the skyrmion width. Our approximation slightly underestimates the energy contribution of exchange energy. Both (a) and (b) shows that Taylor expansion first order gives better result compare to second order. [10]

where K is the anisotropy energy density constant for a given material. The Zeeman energy is

$$E_{zee} = (2\pi t) \cdot BM_s \left[-R^2 - 4\left(1 - \frac{8}{\pi^2}\right)\Delta^2 \right]. \tag{16}$$

Both Equation (15) and (16) agree with the results from Büttner's group. (see Figure 5 and Figure 6, respectively).

Figure 5 is the anisotropy energy per unit area as function of the size (R) and transition width (Δ) of the skyrmion (solid lines). It shows that anisotropy energy linearly increases with the size and the width of the skyrmion. Our anisotropy energy is a another perfect fit with the result from Büttner *et al.* (dashed lines)

Figure 6 shows the Zeeman energy per unit area as a function of size (R) and transition width (Δ) of the skyrmion. It shows that the Zeeman energy decreases with both width and sized of skyrmion. This is because the applied field here is in the same direction as the skyrmion core and so the magnetic field favours the expansion of the skyrmion. Compared with other energy contributes, the Zeeman energy contribution is smaller due to the size $B = 0.001$T used here.

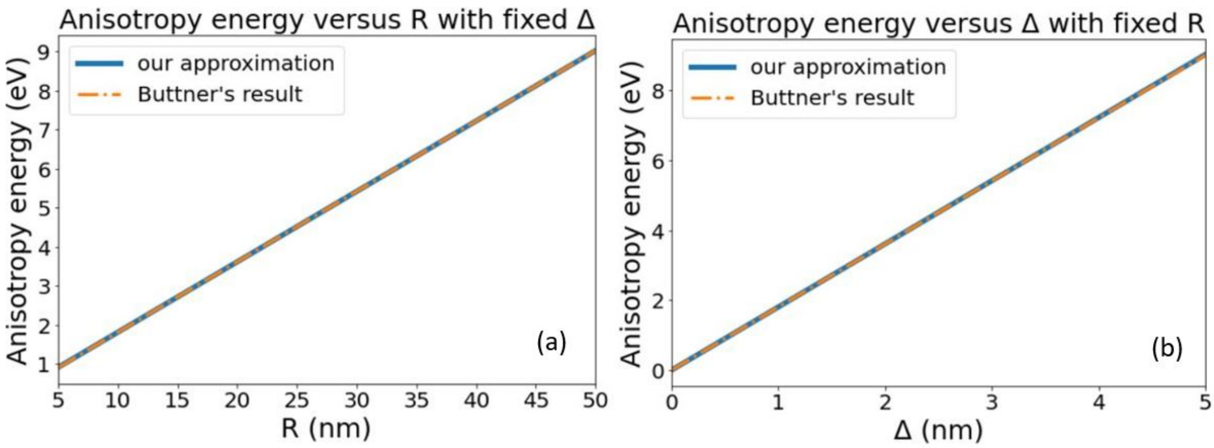

Figure 5: The anisotropy energy per unit area as function of the width and size of the skyrmion: (a) with fixed width $\Delta = 2$nm, anisotropy energy linearly increases with the skyrmion size R; (b) with fixed size R = 20nm, anisotropy energy linearly increases with the skyrmion width Δ. Our predicted results is exact as same as the Büttner's result [10] within the given intervals where skyrmion usually are found.

2.4 Stray-field energy

The stray field energy is the hardest contribution to calculate. The magnetic field outside a magnet is called stray field and within a magnet is known as the demagnetization field. Stray field can be found whenever the magnetization has a component normal to an external or internal surface or nonuniform magnetization.

From Livesey and Davidson's past derivation, the shape of the demagnetization field function actually is similar to a negative magnetization function, which can be approximated by a similar piecewise function, which were used in other energy contribution. [12, 3, 13] The only difference is that the piecewise function profile would be slightly wider and the maximum magnetization would not reach 1. The difference in maximum magnetization is named the δ. The demagnetization field function and δ value, which were calculated by Livesey, were used to develop the expression of demagnetization energy contribution.This produces simple functions of out-of-plane (z) demagnetization energy per unit area and in-plane (ρ) demagnetization energy.

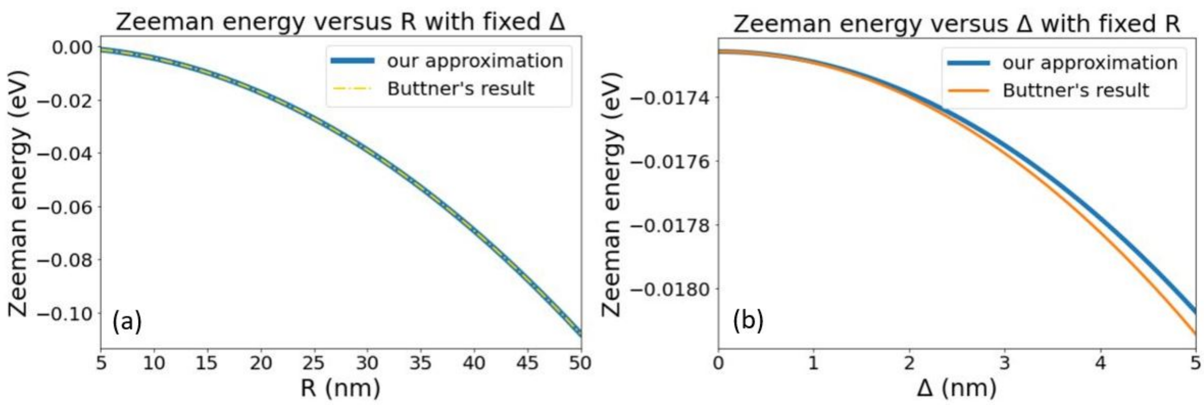

Figure 6: The Zeeman energy per unit area as a function of width and size of the skyrmion: (a) with fixed width $\Delta = 2$nm, the energy decreases with the size of skyrmion R. Our results is identical as Büttner's result [10]; (b) with fixed size R = 20nm, the energy decreases with the width Δ, the energy from our function is almost as same as the Büttner's result [10].The Zeeman energy contribution is smaller compared to other contributions. This is due to the use of a small magnetic field $B = 0.001$T here.

$$E^{z}_{demag} = (2\pi t) \cdot \frac{1}{2}\mu_0 M_s^2 \left[-2R\left(\Delta + \frac{t}{2}\right) - \frac{4}{2}R + t\left(\Delta + \frac{t}{2}\right) + \frac{t\left(\Delta + \frac{t}{2}\right)^2}{R}\left(1 - \frac{4}{\pi^2}\right)\right] \tag{17}$$

$$E^{\rho}_{demag} = (2\pi t) \cdot \mu_0 M_s^2 R\Delta \left(\frac{t}{t + 4\Delta}\right) \tag{18}$$

Equation (17) and (18) are significantly simpler compared to the functions given by Büttner's group, while the values that our functions predict are close to the results of Büttner *et al.*.(see Figure 7 and Figure 8)

3 Energy minimization

In order to do the minimization step, all the energies of different contributors are added together. These are given in the summary Table 1.

The minimization step involves solving for the skyrmion size R and the variation width Δ by differentiating the total energy E and setting the results to zero, namely

$$\frac{\partial E(R,\Delta)}{\partial R} = \frac{\partial E(R,\Delta)}{\partial \Delta} = 0. \tag{19}$$

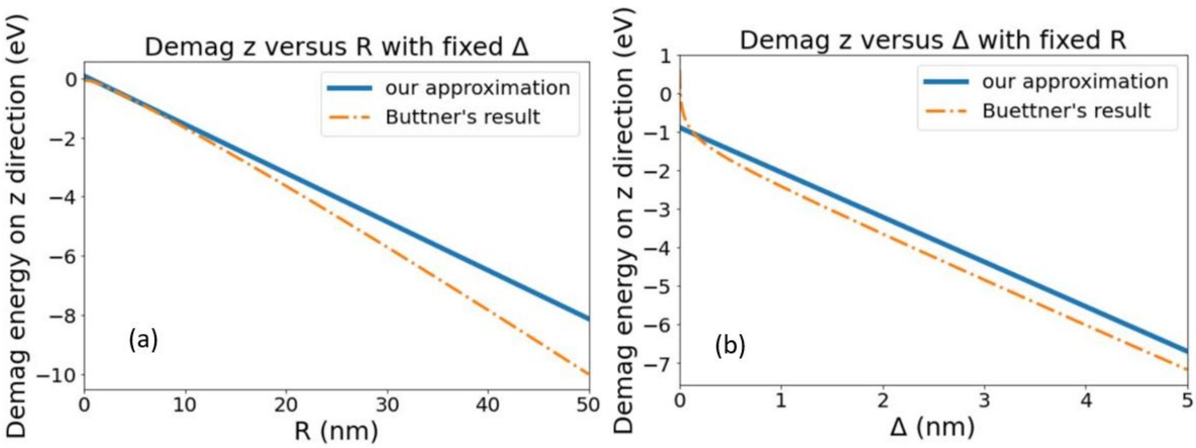

Figure 7: The demagnetization energy (z direction) per unit area as a function of width and size of the skyrmion: (a) with fixed width $\Delta = 2$nm, the energy decreases with the size of skyrmion R; (b) with fixed size R = 20nm, the energy decreases with the width Δ, both results are very close to Büttner's result [10].

Equation (19) was solved when the external magnetic field $B = 0$. It offers decent analytical result for the width and size of a skyrmion on a thin film. The width of the skyrmion as a function of properties of magnetic material is

$$\Delta = \frac{-D\pi + \frac{3}{4}\mu_0 M_s^2 t}{4K + \frac{2\mu M_s^2 t}{t - \frac{\pi D}{K}}} \tag{20}$$

The size of the skyrmion as a function of properties of magnetic material is

$$R = \left| D\pi - \frac{3}{4}\mu_0 M_s^2 t \right| \sqrt{\frac{A}{2\pi^2 A \left[K + \frac{\mu_0 M_s^2 t}{2\left(t - \frac{\pi D}{K}\right)}\right]^2 - \left(D\pi - \frac{3}{4}\mu_0 M_s^2 t\right)^2 \left[K + \frac{\mu_0 M_s^2 t}{2\left(t - \frac{\pi D}{K}\right)}\right]}} \tag{21}$$

When the thickness t in Equations (20) and (21) goes to zero, i.e, when we treat the film as an infinitely thin film, the result agrees with the analytic result from Wang *et al.* [14].

4 Conclusion, Discussion and Future Work

We developed an analytic solution for the size and shape of skyrmions as a function of magnetic material properties. Our results show that the method of using piecewise function to approx-

Table 1: Simplified energy per unit area of each contributors for minimization

Energy contributors	energy per unit area
DMI	$\frac{E_{DMI}}{2\pi R} = D\pi R$
Exchange interaction	$\frac{E_{ex}}{2\pi R} = A\left[\left(\frac{\pi^2}{4}\right)\frac{R}{\Delta} + \frac{2\Delta}{R}\right]$
Anisotropy	$\frac{E_{anis}}{2\pi R} = 2R\Delta K$
Zeeman	$\frac{E_{zee}}{2\pi R} = BM_s\left[-R^2 - 4\left(1 - \frac{8}{\pi^2}\right)\Delta^2\right]$
Stray field / demagnetization (z)	$\frac{E^z_{demag}}{2\pi R} = \frac{1}{2}\mu_0 M_s^2\left(-2R\Delta - \frac{3}{2}Rt + \Delta t\right)$
Stray field / demagnetization (ρ)	$\frac{E^\rho_{demag}}{2\pi R} = \mu_0 M_s^2 R\Delta\left(\frac{t}{4+4\Delta}\right)$

imate the magnetization function is a viable way to simplify mathematical models while still being able to accurately predict experimental results. Comparing our results in Equations (21) and (20) to the analytical result given by Wang *et al.* (need reference), our result contains the actual thickness of the film t. This is more realistic and practical than assuming the magnetic film is vanishingly thick. As shown in Figure 9, the radius or size of the skyrmion blows up when the film is too thick ($t > 3$ nm). This cannot be predicted using a theory that ignores the film's thickness.

Another interesting results by looking into the DMI energy and how it has effect on the size and width of the skyrmion. (see Figure 10) As theoretically predicted, DMI strength is one of the most important energy contribution which plays a major role in stabilizing skyrmions in a thin film. However, when the force of DMI too strong, skyrmions also could not exist on the think film.

Although the analytical result does not involve Zeeman energy contribution, Zeeman energy contribution (see Figure 6) relatively small compared to other energy contribution. Therefore, a small external magnetic field would have a big impact on our analytical result.

For the future work, we would like to compare our result for R and Δ with the result from minimizing the total energy given by Büttner *et al.*. Their functions can not by minimized analytically but only numerically. To minimizing their function requires further work on numerically analysis process. However, their result would be a good bench mark for the accuracy of our analytical result and theory.

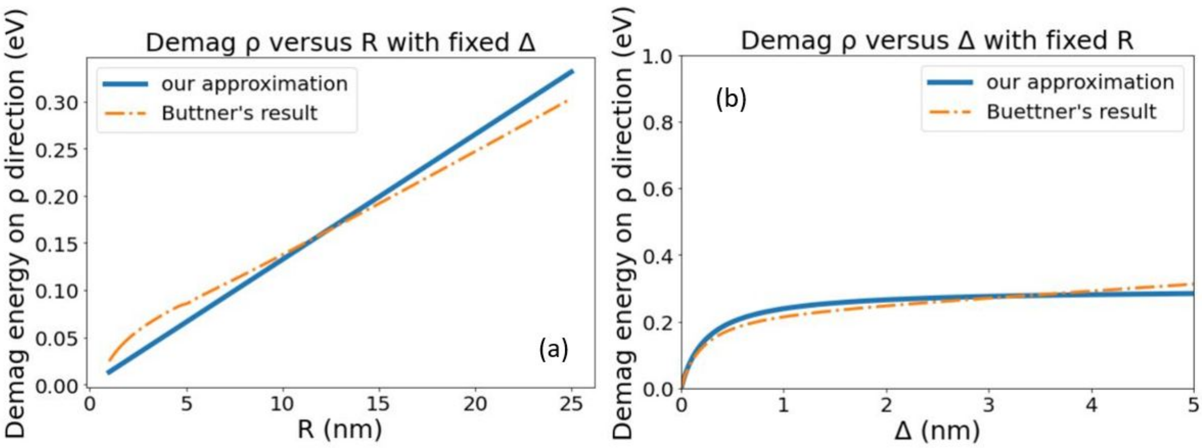

Figure 8: The demagnetization energy (ρ direction) per unit area as a function of width and size of the skyrmion: (a) with fixed width $\Delta = 2$nm, the energy decreases with the size of skyrmion R; (b) with fixed size R = 20nm, the energy decreases with the width Δ, both results are very close to Büttner's result [10].

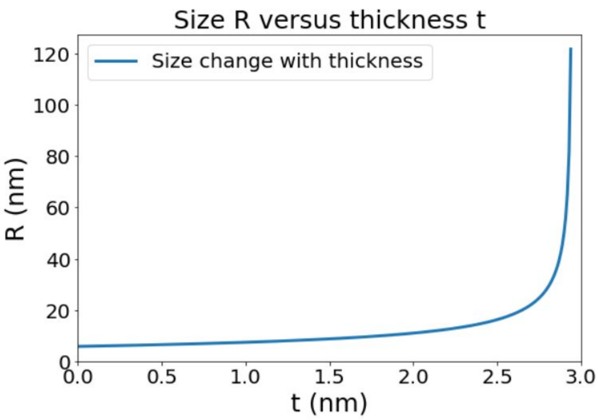

Figure 9: Size of skyrmion as a function of the magnetic film thickness: there is a limitation of thickness for skyrmions to exist in a think film.

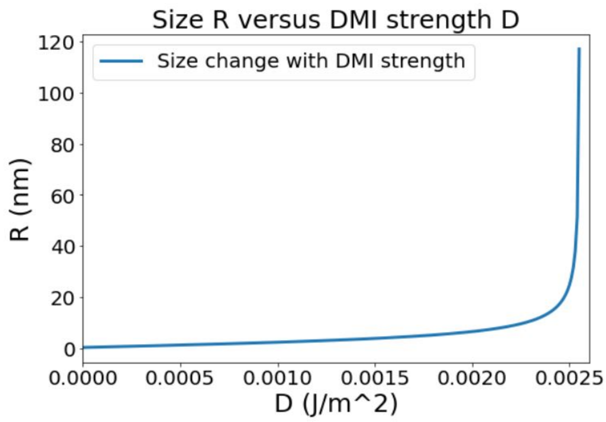

Figure 10: Size of skyrmion as a function of DMI strength: it proves that DMI stabilized the skyrmion, but skyrmions also could not form when DMI energy is too big.

Acknowledgement

The research is based on Karen Livesey's work and under her supervision. I would like to thank her enormous contribution and devotion to this research project and guide me overcome knowledge gaps and through obstacles.

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