AMSI VACATIONRESEARCH SCHOLARSHIPS 2020–21
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Mathematical Modelling of Harmful Algal Blooms

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Vacation Research Scholarships are funded jointly by the Department of Education, Skills and Employment and the Australian Mathematical Sciences Institute.

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Australian Government Department of Education, Skills and Employment

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Abstract:

Nowadays, climate change represents a major threat to life on Earth. These effects affect freshwater and marine environments causing harmful algal. For this reason, it is necessary to develop a model to prevent harmful algal bloom. Our model is based on the basic model for nutrient-phytoplankton interactions, and the model for nutrient dynamics. Then we identify which parameters can be affected by external factors, particularly climate change and how climate can affect these parameters. In this project, these parameters are described by the way mathematically to see the effects of climate change.

1. Introduction:

1.1 Background:

Harmful Algal Blooms (HABs) are characterised as the accumulation and occasional dominance of species, such as phytoplankton, of harmful algae in aquatic systems. Understanding blooms necessitates understanding the evolutionary dynamics of toxic algae, which can be underpinned by mathematical models. Many factors influence the occurrence of HABs, among which nutrient enrichment is widely recognised as the critical cause of HAB formation. Thus, nutrient dynamics play an essential role (Gatz 2019).

Climate change poses an additional challenge since climate controls the factors that regulate algal growth (Paerl et al. 2016). For example, climate change, including global warming, is causing changes to regional rainfall and hydrology, which will have cumulative effects with nutrient-over-enrichment in modulating HABs. Furthermore, an increase in rainfall pattern variability affects the delivery of nutrients and sediments, the exchange of water sediments and metabolism, flushing and residence times and vertical stratification, which in turn can influence HABs domination and persistence.

The temperature is one of the most significant environmental factors affecting phytoplankton physiological processes which occur in various growth and blooms stages. It is typically very harmless, but the right combination of warm water, high amounts of nutrients and a proper mix of sunlight will cause damaging algae to bloom (Climate Central 2017). The prediction is hot and dry in the summer, as the blue-green algae like hot, still water, would possibly boast of blooming. That means that areas of algal bloom usually find this summer longer and more extensive. Warmer temperatures associated with climate change caused nitrogen pollution, so increased nutrient input flow. Yuan et al. (2007, p.646) show an arbitrary rational temperature of the nutrient threshold. Besides, Wells et al. (2015, p.71) state that the earlier spring warming pattern could contribute to the appearance of seed populations in coastal surface waters through the earlier emergence of permissive flowering temperatures and increased flowering rates in higher temperatures

1.2 Report structure:

The following report document is being organised. Part 2 introduces the equation for temperature and time. Part 3 gives two models which are mentioned before to identify how climate change affects the harmful algal bloom. Last but not least, part 4 gives the conclusion and identifies the future research direction.

1.3 Statement of Authorship:

The concept and model description for the project was developed by Zhang. Ngoc Phuong Van Nguyen has written the model based on the studies of Amit Huppert, Bernd Blasius and Lewi Stone. Nguyen and Zhang conducted analysis and evaluation of the findings. Zhang gave instructions and supervision. AMSI and the Australian Ministry of Education provided funds for the project

2. Equation for temperature and time:

According to our prediction, changes in temperature have the potential to affect nutrient. Therefore, the first thing to determine is the change in temperature each year. According to Mackintosh (2001), the sinusoidal functions can be used to model year-round temperature variations in the given city:

$$T = m\sin[n(t-p)] + q \qquad (1)$$

where T represents the temperature (◦C), t is the month of the year with January as 1 and m, n, p and q: constants with m is the amplitude:

$$m = \frac{1}{2}(high\ temp - low\ temp)$$

The temperature repeats each year, and the period is 12 months, so:

$$n = \frac{2\pi}{12} = \frac{\pi}{6}$$

Because the maximum value of an unshifted sine curve with period 12 occurs at $\frac{1}{4} \times 12 = 3$ units from the start of the cycle, p can be determined:

$$p = hottest\ month - 3$$

q is the vertical shift with:

$$q = \frac{1}{2}(high\ temp + low\ temp)$$

This equation can help determine the average temperature per month of the year, from which we will make the appropriate changes for harmful algal blooming.

3. Assumptions of the model:

3.1 Model A: nutrient-phytoplankton model

The model consists of three variables: nutrient levels N, phytoplankton biomass P, and temperature T. Small nutrient amounts are supposed to enter the environment at a slow but constant rate and we first attempt to establish the impacts of phytoplankton dynamics of those nutrients (Huppert et al. 2002, p.158). Phytoplankton, P, depends on growth nutrients N and is removed by mortality and sinking from the water-column. Besides, there is an assumption that temperature affects nutrient supply N over the time. It gives the following system:

$$\dot{N} = a - bNP - eN + \dot{T} \quad (2)$$

$$\dot{P} = cNP - dP \quad (3)$$

From Equation (1), the derivation of temperature is determined:

$$\dot{T} = mn\cos[n(t-p)] \quad (4)$$

There are five parameters in this first model. Nutrient supply at a constant rate, a and nutrient uptake rates of phytoplankton are determined by the parameters b and c. Dynamics of uptake from Michaelis-Menten may be able to reflect the nutrient dynamics more realistically ((Huppert et al. 2002, p.159). The parameters d and e are the loss rates of phytoplankton and nutrients, respectively, which we have taken to be constant here.

3.2 Model B: the effect of climate change on parameter

The absolute uptake rate was determined as follow: bNP and cNP where NP is the particulate nutrient in the sample. In plastic Michaelis–Menten model (Yu et al. 2018), nutrient uptake rate is expressed as:

$$b = V = \frac{V_{max}[S]}{[S]+K_S} \quad (5)$$

In this equation, $[S]$ is the nutrient concentration, V_{max} is the maximum uptake rate that is describe as:

$$V_{max} = nh^{-1} \quad (6)$$

And K_S is the half saturation constant (an index of the affinity to NH_4^+, HNO_3^-) that is expressed as:

$$K_s = (Avh)^{-1} \quad (7)$$

In Eq.(6) and Eq.(7), h is known as the handling time, A is area of one uptake site, and v is a constant mass transfer coefficient:

$$v = \frac{D}{r} \tag{8}$$

where D is molecular diffusion, and r is cell radius.

Substituting Eq.(6) and (7) into Eq.(5), we have:

$$V = \frac{nh^{-1}[S]}{[S]+(Avh)^{-1}} \tag{*}$$

From Eq.(*), the uptake rate V reaches a maximum rate with the rise in the nutrient concentration $[S]$. Therefore, the uptake level is restricted by processing time h and number of sites per cell n at high concentrations of nutrient, which can be assumed to be a purely biological restriction.

Aksnes and Egge (1991) assumed that the biological reaction rates deciding handling time h are exponentially specified in the temperature related Q_{10} concept:

$$h^{-1} = h_0^{-1}\exp(gT) \tag{9}$$

In this equation, h_0 is handling time per ion at 0◦C, and $g = \frac{lnQ_{10}}{10}$.

Substituting Eq.(9) into Eq.(6), we get:

$$V_{max} = nh_0^{-1}\exp(gT)$$

This is shown that V_{max} increases exponentially with temperature.

K_s are determined not only by the time it is handled but also molecular diffusion D depending on the temperature, which acts through the mass transfer coefficient. Combining Eq.(8), Eq.(9) and Eq.(7), we get:

$$K_S = \left(Ah_0rD(T)\right)^{-1}\exp(gT)$$

where $D(T)$ is the temperature dependence of molecular diffusion. This equation show K_s increases with temperature.

4. Conclusion:

Overall, we have introduced two model that show the harmful agal bloom is result of climate change. Model A has three equations with the first two are equation expressed as phytoplankton bloom and the 3rd equation is for the temperature. In this model, I incorporate the 3rd equation into these equations to reflect the effect of climate change on the nutrient dynamics. In the second model, we identify which parameters that could affect the nutrient supply can be affected by climate change and Identify how climate change can affect these parameters. In particularly, the maximum uptake rate and the half saturation constant are the key variables of the nutrient supply and both of them also increase with temperature. For this reason, climate change has an important role for blooming harmful alga. Besides, we can combine these models with the function temperature and time to predict the affected temperature in any month of the year to have timely preventive measures.

In the future, we aim to numerically investigate the resulting system that can be done by Euler's method. This helps to illustrate the effects of climate change/temperature on the nutrient concentration and phytoplankton concentration.

Acknowledgements:

I would like to acknowledge Associate Professor Tonghua Zhang for his assistance and guidance during this entire semester. Professor Tonghua Zhang significantly helped me to develop my mathematical analysis skills. He introduced me to the projects that lead to this investigation and caught me up on background theory with great attention to detail. His direction has been thorough but not restrictive - allowing me the opportunity to explore this topic independently at times, but not leaving me without guidance.

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